Fast Algorithms for Synchrophasor Computations

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Real-time security monitors @ WSU

Oscillation Monitoring System

TVA, Entergy, Idaho Power

System Security Status
Real-time Display & Control

PMUs & PDC
PMU Real-time data

Voltage Stability Monitor
Entergy, Idaho Power

Angle Stability Monitor
PMU Applications

- The computational complexity of PMU data based algorithms is not scalable as such.
  - # of PMUs ↑, Size of the matrix ↑
  - Large-scale dense matrix. Information sparse.

- Scalable solutions needed with urgency.
  - Custom designed scalable algorithms
  - Tailor existing algorithms to parallel platforms
  - New distributed algorithms
Small Signal Stability

- Oscillations must remain well-damped for stability
- Either sustained oscillations or growing oscillations called small-signal instability
- Caused by unusual operating conditions or poor control designs
- Some eigenvalues become negatively damped resulting in small signal instability
- August 10, 1996 WECC blackout a classical example
Results from Two Engines

Event Analysis

1.2 Hz at +1.5% damping. Local Mode.

Ambient Noise Analysis

1.2 Hz at +1.8% damping. Local Mode.

Nov. 29th 2007 event
Complementary Engines

- Event Analysis Engine (EAE)
  - Multiple algorithms
  - Prony, Matrix Pencil, HTLS, and ERA
  - Aimed at events resulting in sudden changes in damping

- Damping Monitor Engine (DME)
  - Ambient noise based. Continuous. Provides early warning on poorly damped modes.
  - Frequency Domain Decomposition (FDD), Stochastic Subspace Identification (SSI)
Results from Two Engines

- Event Analysis: 1.2 Hz at +1.8% damping. Local Mode.
- Ambient Noise Analysis: 1.2 Hz at +1.5% damping. Local Mode.

Nov. 29th 2007 event
Frequency Domain Decomposition (FDD)

- Collect and preprocess signals from PMUs
- Power spectrum estimation by FFT and Multi-Taper Method
- Apply SVD on the power spectrum matrix
  - Approximate the largest singular value by the trace of the power spectrum matrix (Fast FDD)
- Apply inverse FFT on largest singular values
- Extract the pole frequency and damping ratio from the exponential form by ringdown analysis
SVD in PMU Applications

- SVD ➔ Fundamental feature extraction algorithm in many PMU applications

- Oscillation monitoring methods
  - Eigenvalue Realization algorithm (ERA)
  - Matrix Pencil algorithm
  - Hankel Total Least Square (HTLS) algorithm
  - Total least square estimation of signal parameters via rotational invariance techniques (TLS-ESPRIT)
  - Extended complex Kalman filter (ECKF)
  - Stochastic Subspace Identification (SSI) method

- Real-time voltage stability monitoring using PMUs, and PMU placement and ranking problem
SVD problem in SSI

SSI-COV method

\[ Y_p = \begin{bmatrix} y_0 & y_1 & \cdots & y_{J-1} \\ y_1 & y_2 & \cdots & y_J \\ \vdots & \vdots & \ddots & \vdots \\ y_{J-1} & y_J & \cdots & y_{I+J-2} \end{bmatrix} \in \mathbb{R}^{J \times J}, \]

\[ Y_f = \begin{bmatrix} y_I & y_{I+1} & \cdots & y_{I+J-1} \\ y_{I+1} & y_{I+2} & \cdots & y_{I+J} \\ \vdots & \vdots & \ddots & \vdots \\ y_{2I-1} & y_{2I} & \cdots & y_{2I+J-2} \end{bmatrix} \in \mathbb{R}^{J \times J}. \]

\[ H = Y_f Y_p^T = O_I G, \]

\( J \) is known as initial window

\( I \) is a user defined parameter

Each \( y_k \) in these two matrices is a vector of size \( I \).

where \( I \) is the \# of PMU signals

\[ 1 \times 1 \]
**SVD problem in SSI**

2I → 5 to 20 seconds × Sampling rate of 30 Hz

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<th>2I</th>
<th># of Signals</th>
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Fast SVD Algorithms

- Two fast SVD approaches have been used for speeding up PMU data based algorithms
  - Randomized SVD method
  - Augmented Lanczos Bidiagonalization method
Angle Stability Detection

Area 1  P  Area 2

Low power transfer

Rotor phase angles stay together
Transient unstable

Heavy power transfer

Rotor phase angles separate away
Critical Questions

- Angle instability phenomenon?
- Which area is accelerating?
- Which area is decelerating?
- Generator tripping? Which generator?
- Load shedding? Which load?
- All decisions made using real-time wide-area phase angle and frequency measurements.