

# Scaling log-linear analysis to datasets with 1,000+ variables

François Petitjean and Geoff Webb



2015 SIAM International  
Conference on **DATA MINING**



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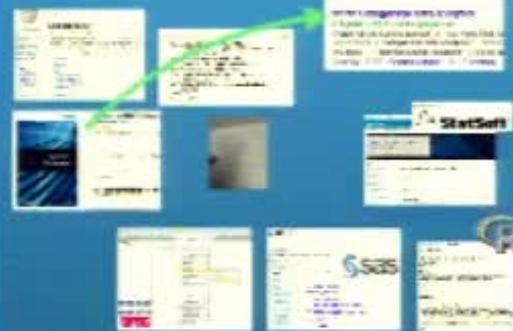
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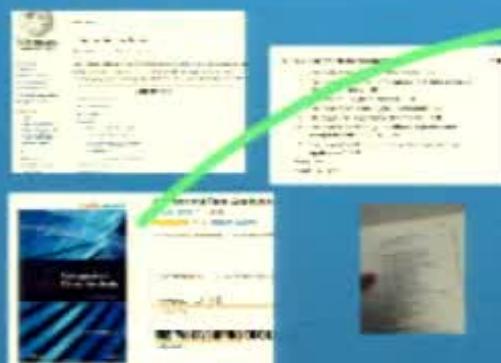
# Motivation



Log-linear analysis = one of THE standard methods  
in statistics

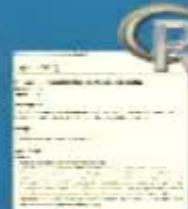
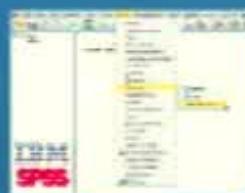


Log-linear analysis = one of THE standard methods  
in statistics

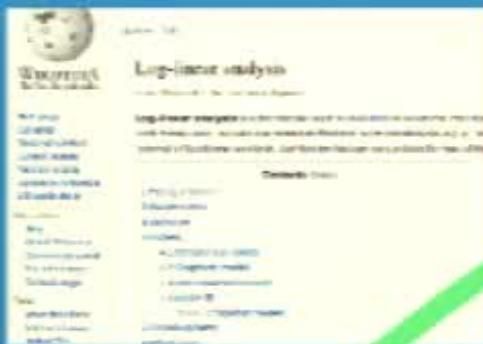


#### more Categorical data analysis

A Agresti - 2012 - statatutorial.com  
Value to the Second Edition! An in-depth look at  
log-linear analysis & interpretation uses examples... 1900+  
Pages! ---Please contact Dennis! If you want  
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# Log-linear analysis = one of THE standard methods in statistics

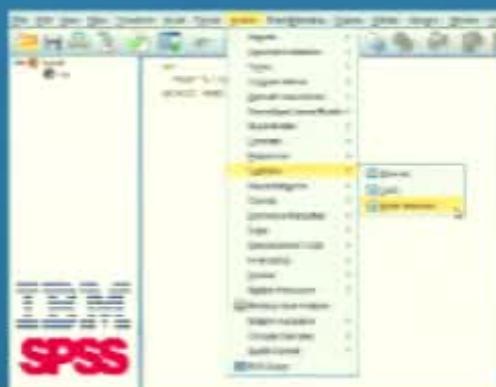
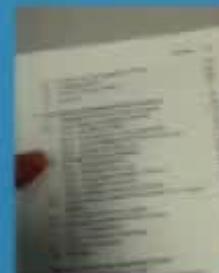
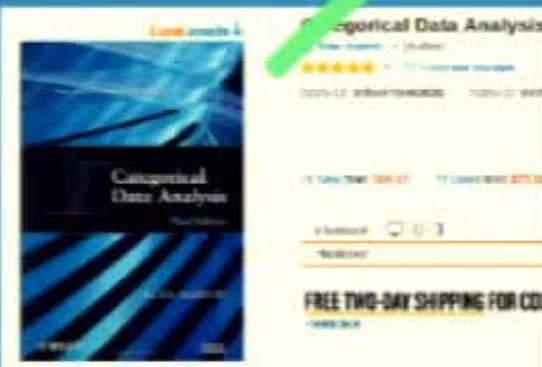


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## [BOOK] Categorical data analysis

A Agresti - 2013 - books.google.com

Praise for the Second Edition "A must-have book for applications in **categorical data analysis**."—Statist this book."—*Pharmaceutical Research*" If you do an Cited by 17177 Related articles All 27 versions



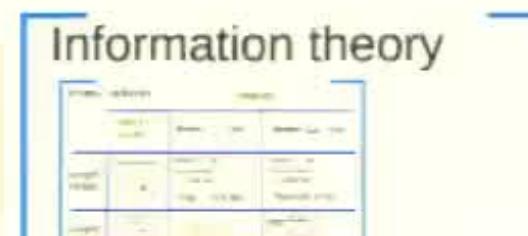
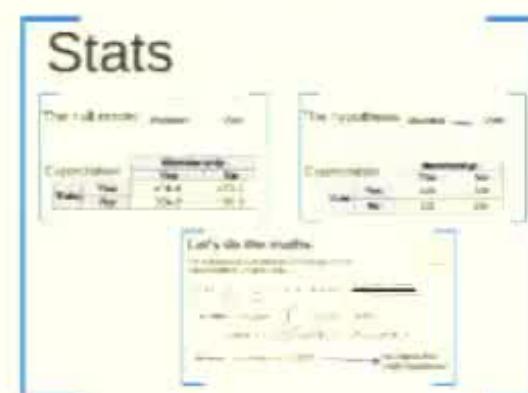
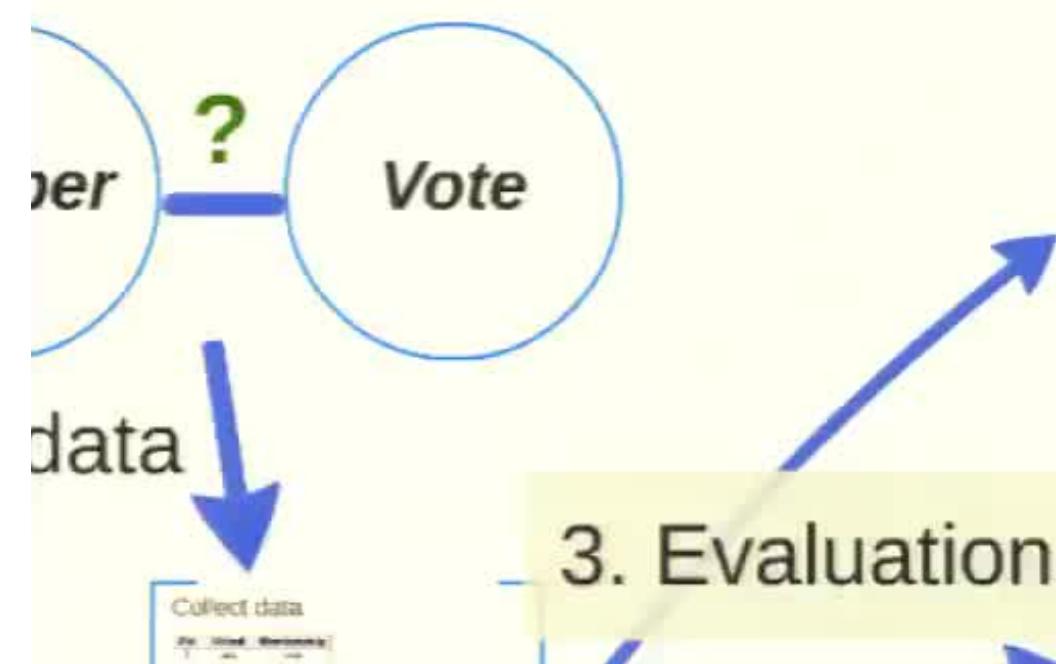
# near analysis

*...a very simple example*

"Is there a difference in voting behavior for the presidential election between people who are members of being a **member of an organization**?"



.. formulate hypotheses

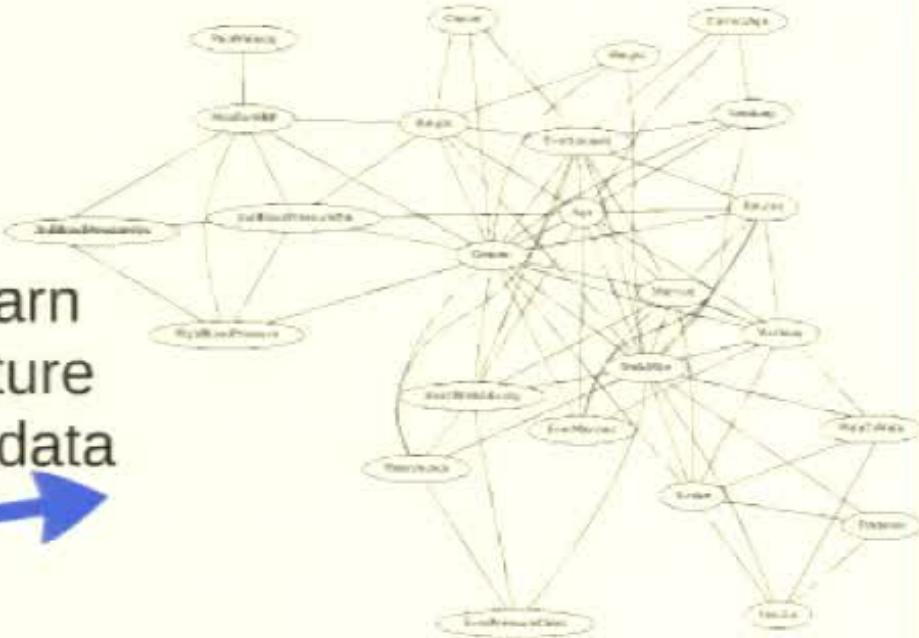


# Log-linear analysis

## Learning graphical models from data

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Gender	Age	Working	Retire									Cancer	Diabetes	Insulin
2	Male	55-64	No	Yes									No	No	No
3	Female	55-64	No	No									No	No	No
4	Male	55-64	Yes	Yes									No	No	No
5	Female	65-74	No	Yes									No	No	No
6	Female	65-74	Yes	No									No	No	No
7	Female	75-79	No	Yes									No	No	No
8	Female	75-79	Yes	No									No	No	No
9	Male	75-79	No	Yes									No	No	No
10	Female	75-79	No	Yes									No	No	No
11	Male	75-79	Yes	No									No	No	No
12	Female	75-79	Yes	No									Yes	No	No
13	Male	80-84	No	Yes									No	No	No
14	Female	80-84	No	Yes									No	No	No
15	Male	75-79	No	Yes									No	No	No
16	Female	80-84	No	Yes									No	No	No
17	Male	75-79	No	Yes									No	No	No
18	Male	75-79	Yes	No									No	No	No
19	Male	75-79	No	Yes									No	No	No
20	Male	75-79	No	Yes									No	No	No
20981	Male	70-74	No	Yes									No	No	No
20982	Female	70-74	Yes	Yes									No	No	No
20983	Female	70-74	No	Yes									No	No	No
20984	Female	70-74	No	Yes	No								No	No	No
20985	Female	70-74	No	Yes	No								No	No	No
20986	Male	70-74	No	Yes	Suspect								Suspect	No	No
20987	Female	70-74	No	Yes	No								No	No	No
20988	Male	70-74	Yes	No									No	No	No

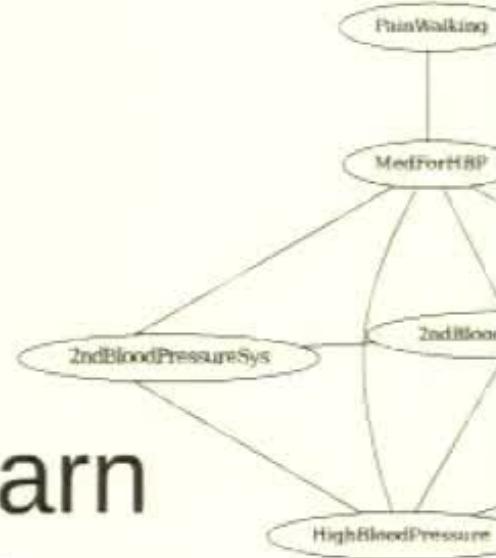
1. Learn  
structure  
from data



2. Use the  
structure



	A	B	E	P	GT	GU	GV	GW
1	Gender	Age	Working	Retire	Smoker	Cancer	Diabetes	Insulin
2	Male	85over	No	Yes	No	No	No	No
3	Female	85over	No	No	No	No	No	No
4	Male	85over	?	?	No	Yes	No	No
5	Male	80-84	No	Yes	No	No	No	No
6	Female	80-84	No	No	No	No	No	No
7	Female	85over	No	Yes	No	No	No	No
8	Female	80-84	No	No	No	No	No	No
9	Male	80-84	No	Yes	No	No	No	No
10	Female	80-84	No	Yes	No	No	No	No
11	Male	80-84	No	Yes	No	No	No	No
12	Female	75-79	No	No	No	Yes	No	No
13	Male	80-84	Yes	Yes	No	No	No	No
14	Female	80-84	No	Yes	No	No	No	No
15	Male	75-79	No	Yes	No	No	No	No
16	Female	80-84	No	Yes	No	No	No	No
17	Male	75-79	No	Yes	No	No	No	No
18	Male	75-79	Yes	No	Suspect	No	Suspect	No
19	Male	80-84	No	Yes	No	No	No	No
20	Female	75-79	No	Yes	No	No	No	No
99981	Male	70-74	No	No	No	Yes	No	No
99982	Female	70-74	Yes	Yes	No	No	No	No
99983	Female	70-74	No	Yes	No	No	No	No
99984	Female	70-74	No	Yes	No	No	No	No
99985	Female	70-74	No	Yes	No	No	No	No
99986	Male	70-74	No	Yes	Suspect	No	Suspect	No
99987	Female	70-74	No	Yes	No	No	No	No
99988	Male	70-74	Yes	No	No	No	No	No



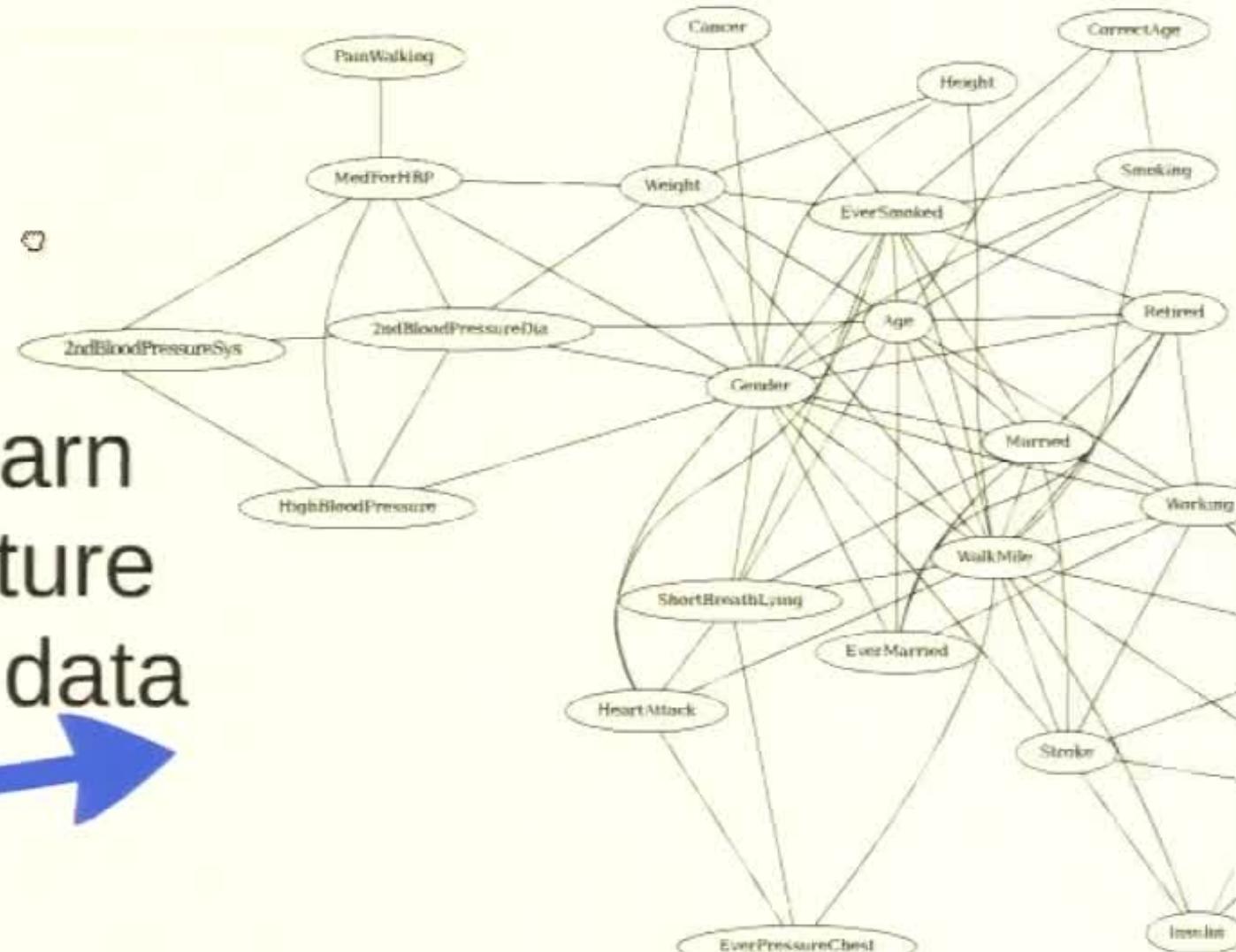
1. Learn structure from data



# Physical models from data

U	GV	GW
cer	Diabetes	Insulin
No	No	
Suspect	No	
No	No	
Suspect	No	
No	No	

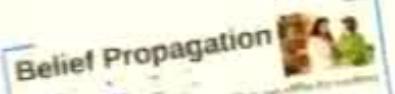
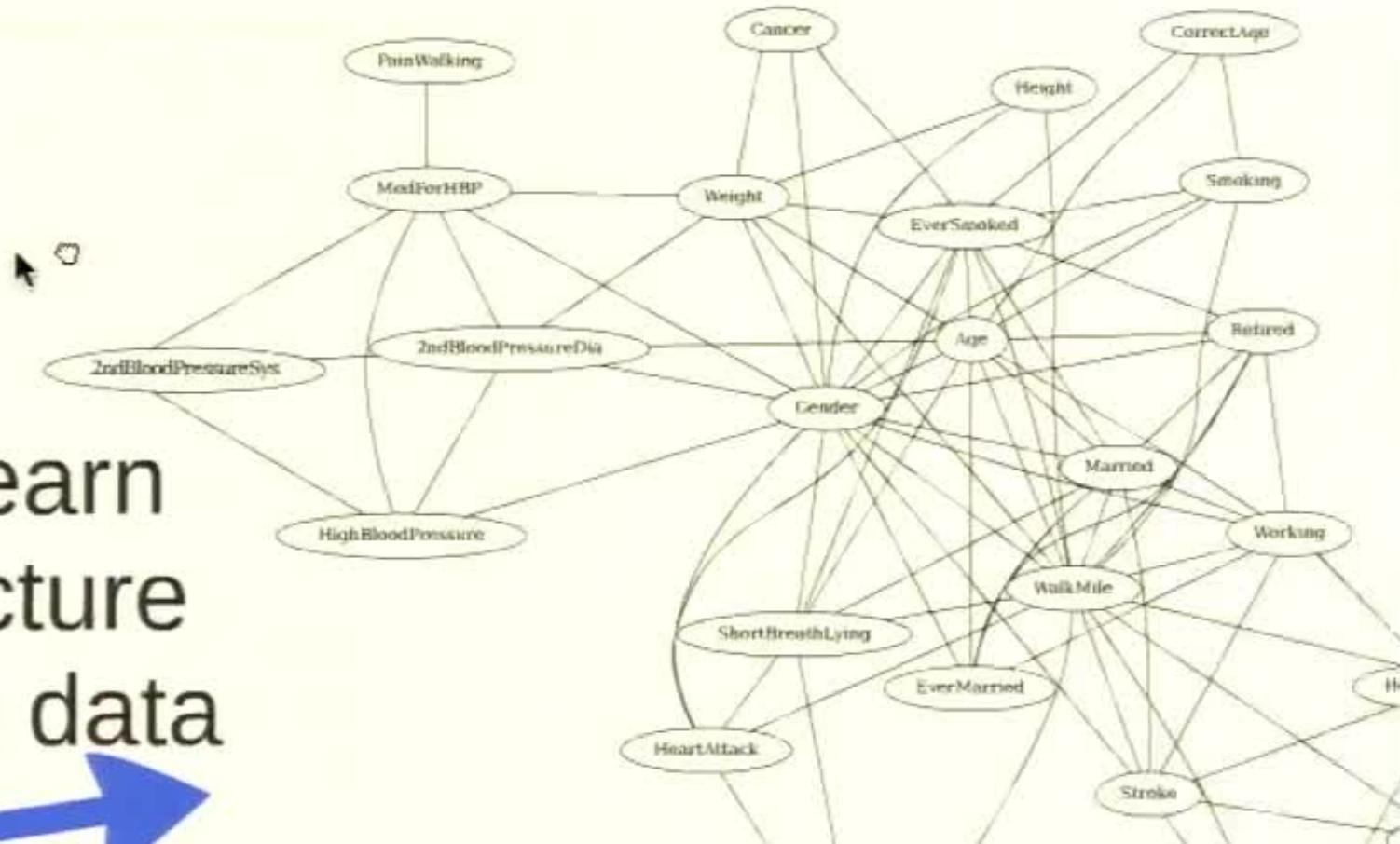
1. Learn  
structure  
from data



# Graphical models from data

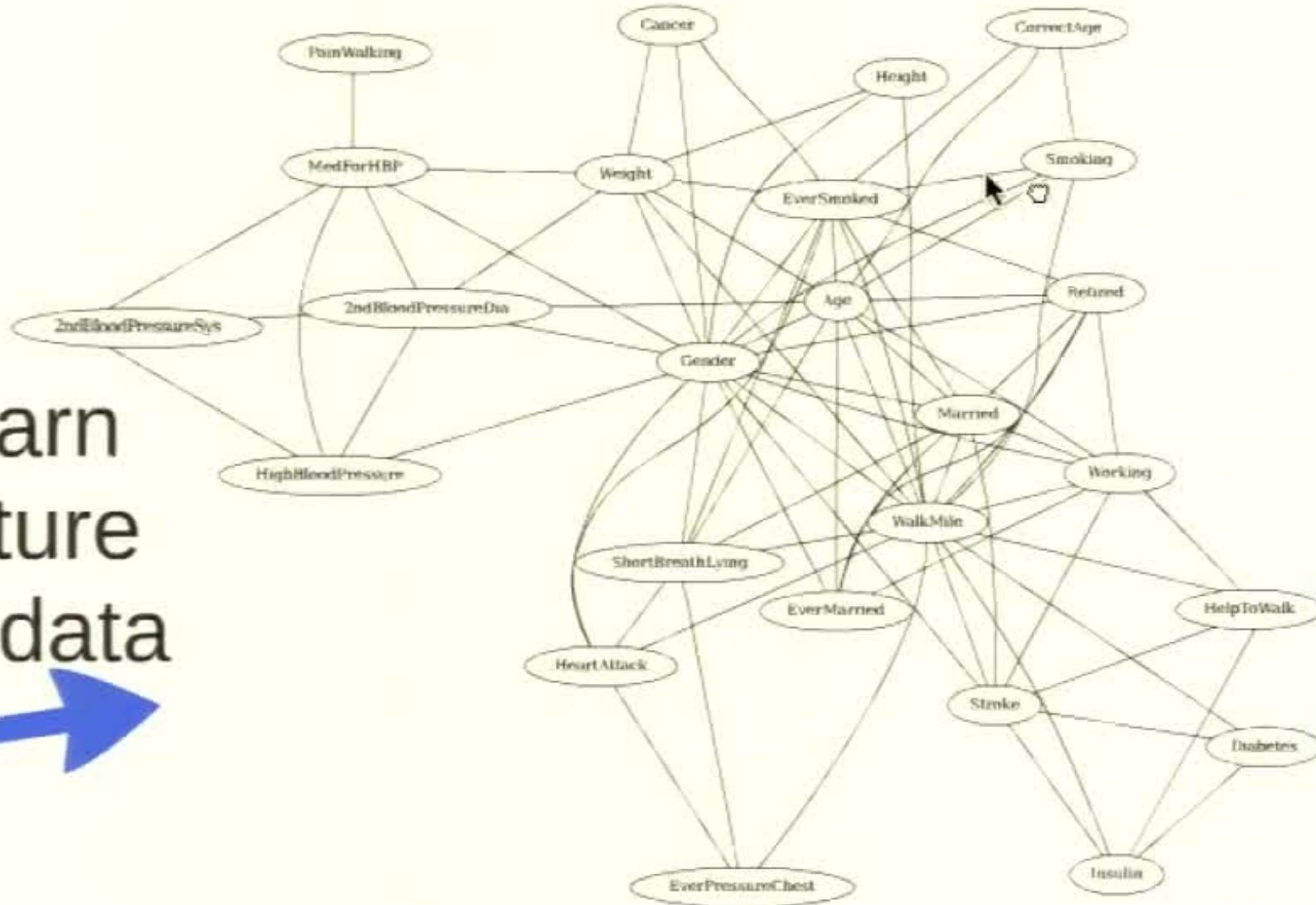
1. Learn  
structure  
from data

GV	GW
Diabetes	Insulin
No	No
Suspect	No
No	No
Suspect	No
No	No

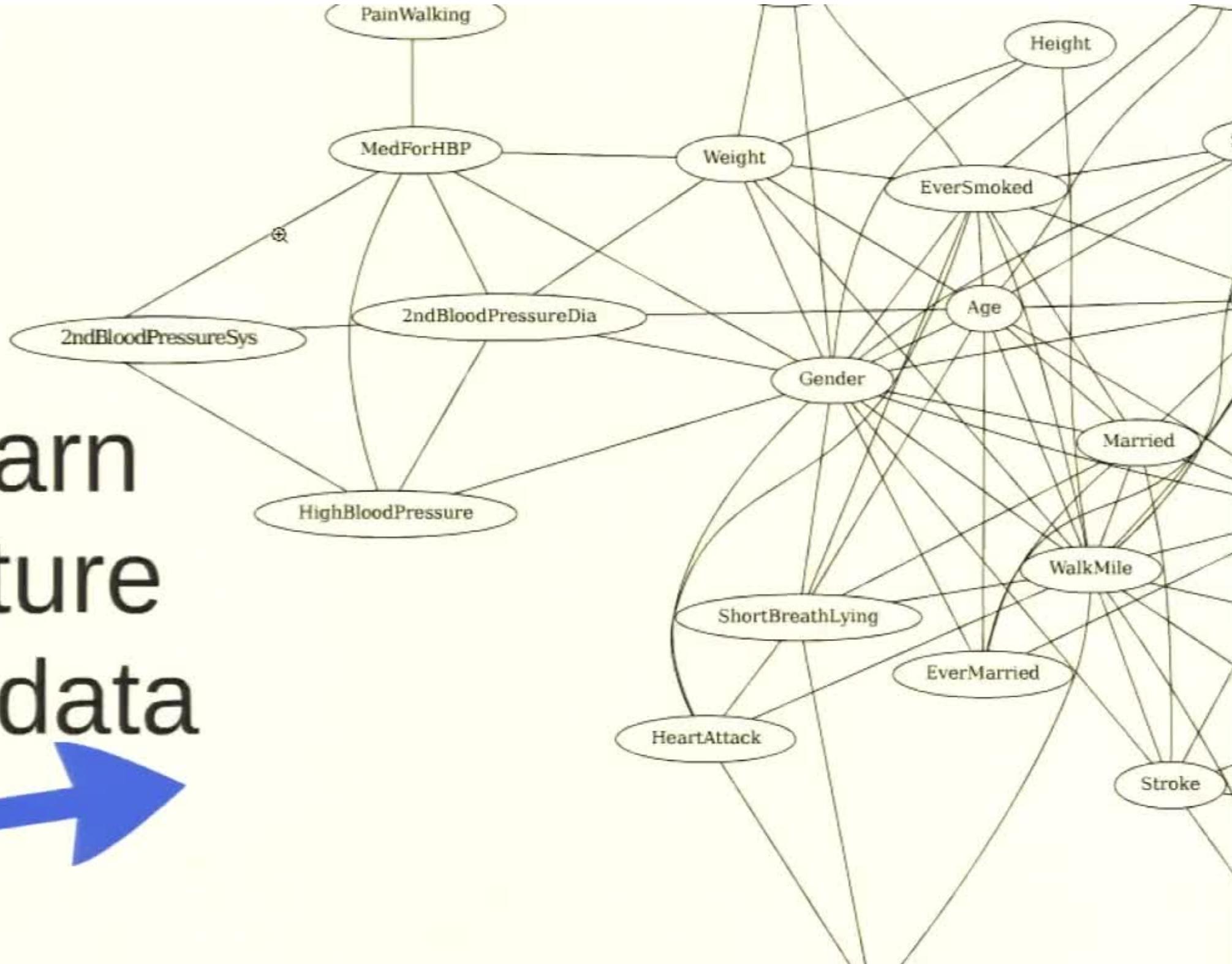


# cal models from data

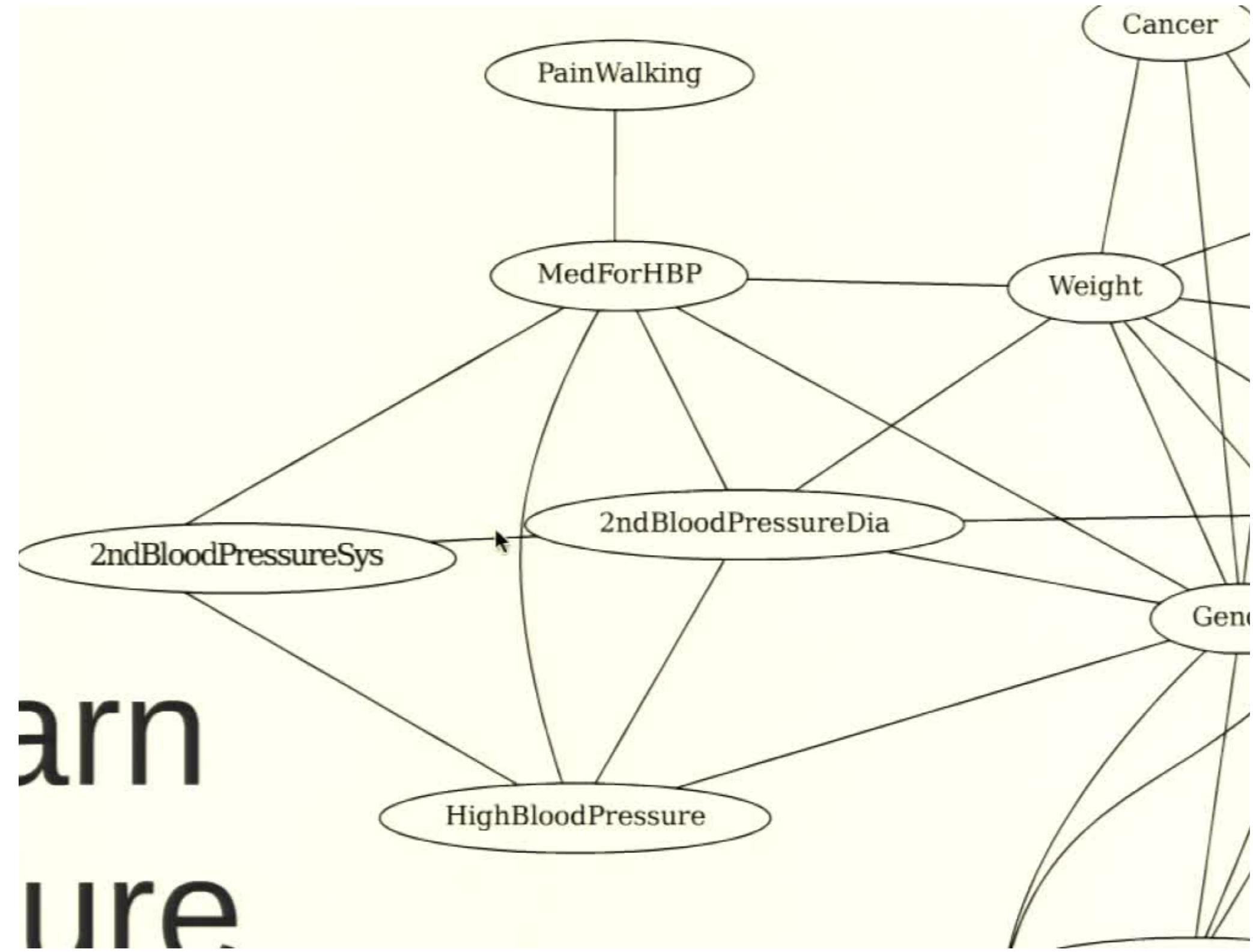
1. Learn  
structure  
from data



Learn  
future  
data



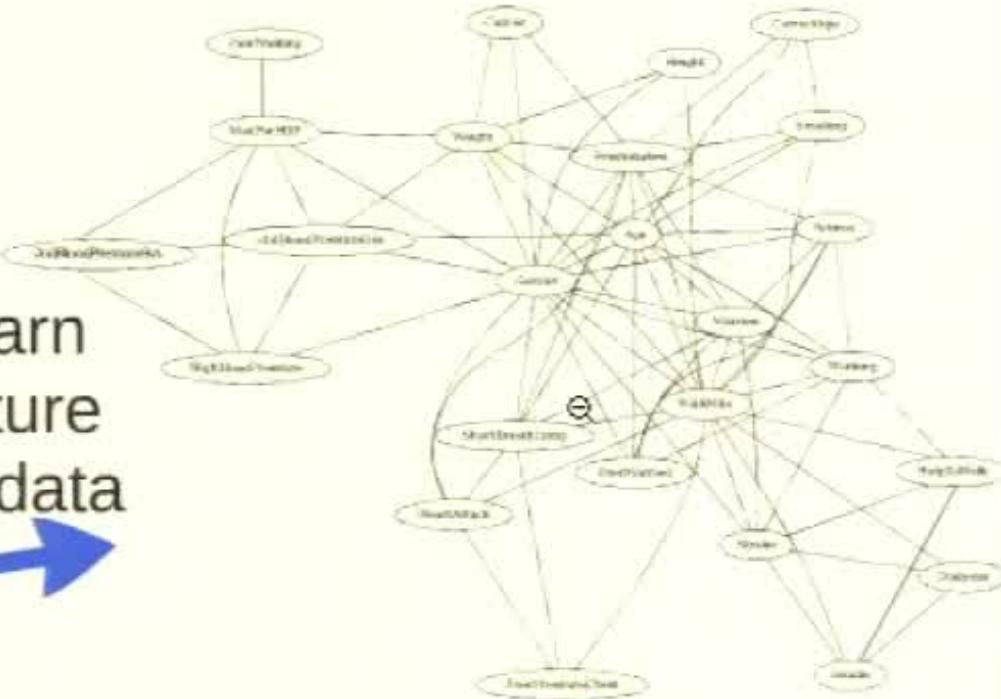
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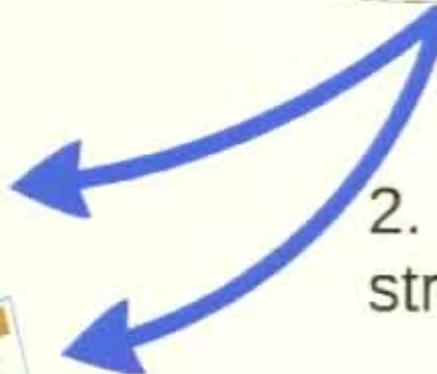
# Pg-linear analysis learning graphical models from data

	A	B	E	R	ST	GU	CV	GW
1	Gender	Age	Working	Retire	Take	Cancer	Diabetes	Insulin
2	Male	55over	No	Yes	No	No	No	No
3	Female	55over	No	No	No	No	No	No
4	Male	55over	??	?	No	Yes	No	No
5	Male	50-64	No	Yes	No	No	No	No
6	Female	50-64	No	No	No	No	No	No
7	Female	55over	No	Yes	No	No	No	No
8	Female	50-64	No	No	No	No	No	No
9	Male	50-64	No	Yes	No	No	No	No
10	Female	50-64	No	Yes	No	No	No	No
11	Male	50-64	No	Yes	No	No	No	No
12	Female	75-79	No	No	Yes	No	No	No
13	Male	60-64	Yes	Yes	No	No	No	No
14	Female	50-64	Yes	Yes	No	No	No	No
15	Male	75-79	No	Yes	No	No	No	No
16	Female	50-64	No	Yes	No	No	No	No
17	Male	75-79	No	Yes	No	No	No	No
18	Male	75-79	Yes	No	Suspect	No	Suspect	No
19	Male	50-64	No	Yes	No	No	No	No
20	Male	75-79	No	Yes	No	No	No	No
99981	Male	70-74	No	No	No	No	No	No
99982	Female	70-74	Yes	Yes	No	No	No	No
99983	Female	70-74	No	Yes	No	No	No	No
99984	Female	70-74	No	Yes	No	No	No	No
99985	Female	70-74	No	Yes	No	No	No	No
99986	Male	70-74	No	Yes	Suspect	No	Suspect	No
99987	Female	70-74	No	Yes	No	No	No	No
99988	Male	70-74	Yes	No	No	No	No	No

1. Learn structure from data

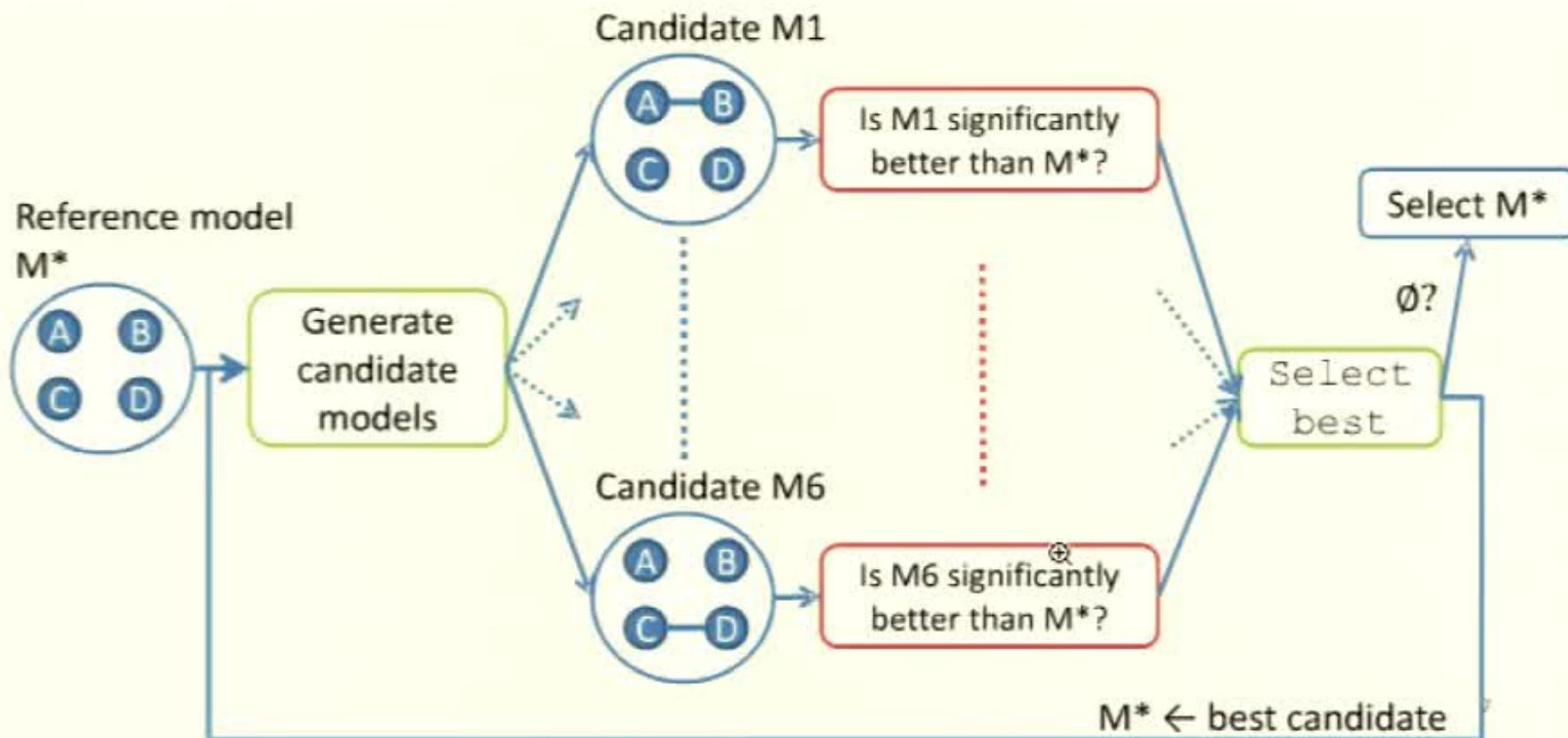


2. Use the structure



# Log-linear analysis for more than 2 variables

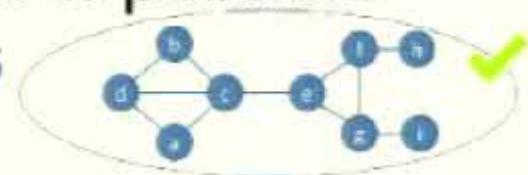
*General framework: selecting a statistically significant log-linear model = superset of Markov Networks*



## What have shown that...

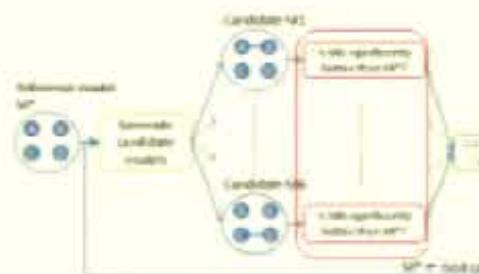
Scalability to datasets with 100 variables is possible for the class of **decomposable models**

- ICDM 2013: statistical tests
- ICDM 2014: minimum description length



**Limitation:** process quadratic with the number of variables

→ 1,000+ variables => **several days** of computation



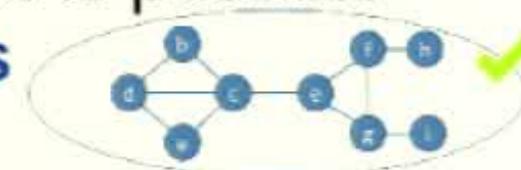
What this paper shows:

We can gain **4 orders of magnitude** while getting **exactly the same results.**

→ 1,000+ variables => **1 minute** of computation

## What have shown that...

Scalability to datasets with 100 variables is possible for the class of **decomposable models**



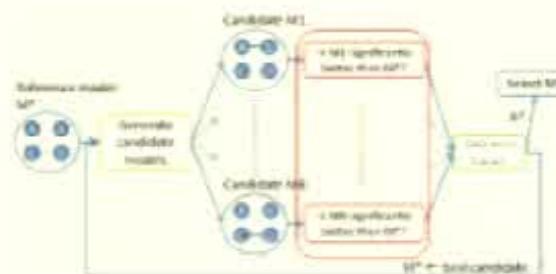
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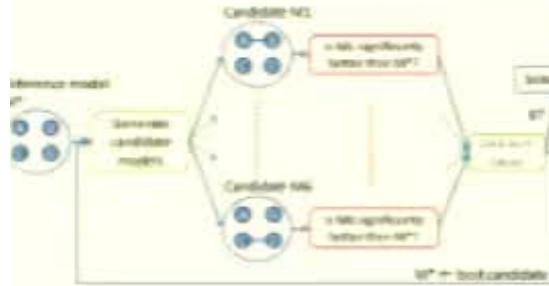
# zed lysis

## Running times



### g-linear analysis for more than 2 variables

General framework: selecting a statistically significant linear model = superset of Markov Networks



### What have shown that...

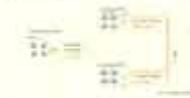
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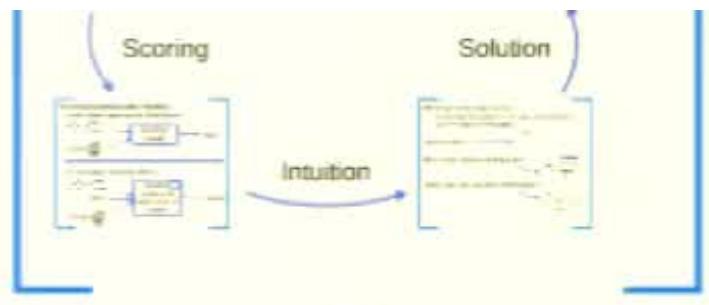
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What this paper shows:

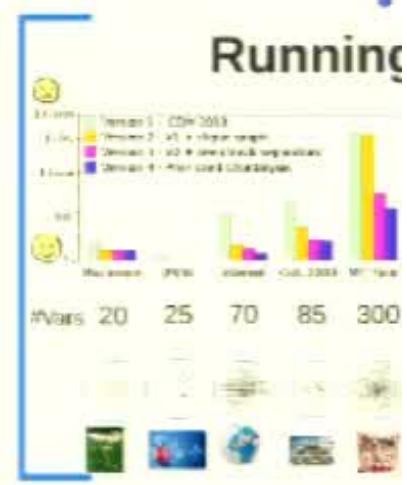
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→ 1.000+ variables => 1 minute of computation

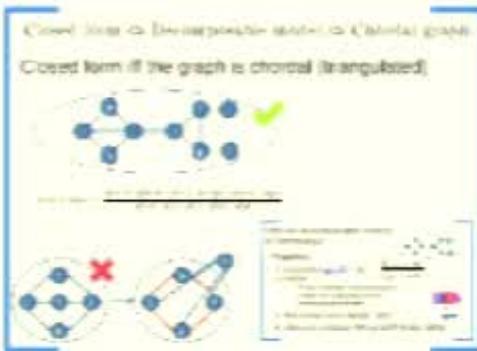


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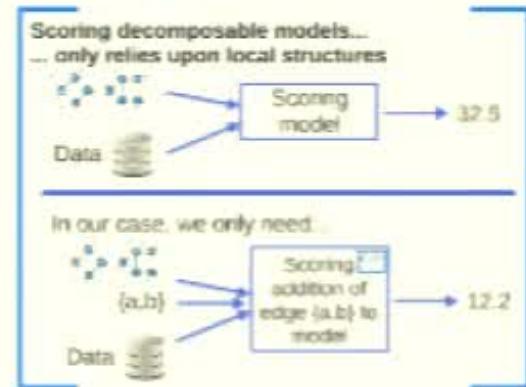
# Prioritized Chordalysis



# Prioritized Chordalysis

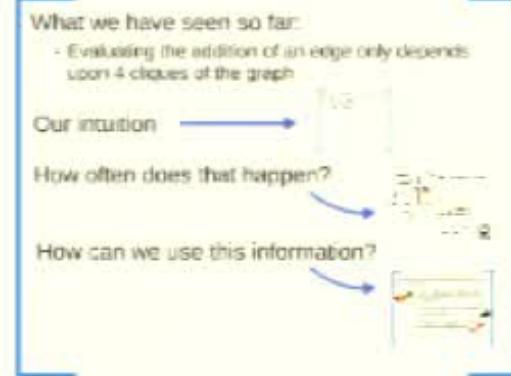


Scoring



Intuition

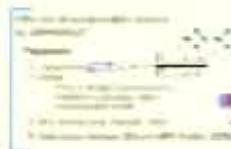
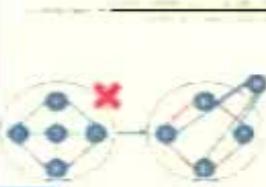
Solution



# Prioritized Chordal

Closed form  $\Rightarrow$  Decomposable model  $\Rightarrow$  Chordal graph

Closed form iff the graph is chordal (triangulated)



## Scoring

Scoring decomposable models...  
... only relies upon local structures



Scoring model

→ 32.5

In our case, we only need...



Scoring addition of  
edge  $\{a,b\}$  to  
model

→ 12.2

## Solution

What we have seen so far:

- Evaluating the addition of an edge on upon 4 cliques of the graph

Our intuition →

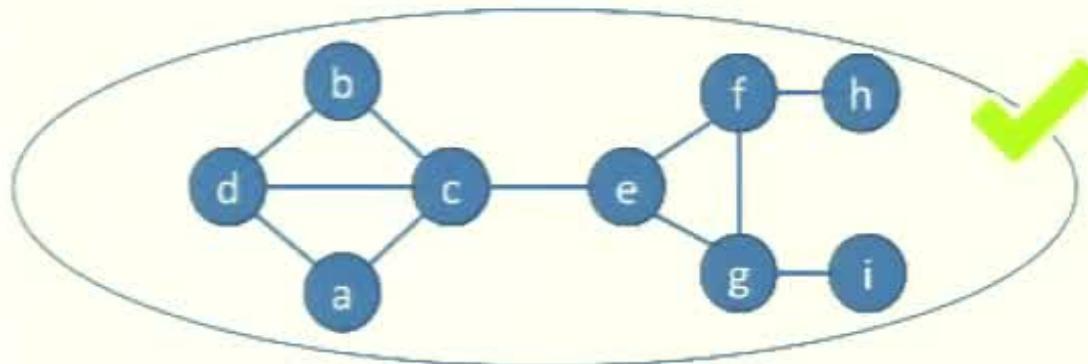
How often does that happen?

How can we use this information?

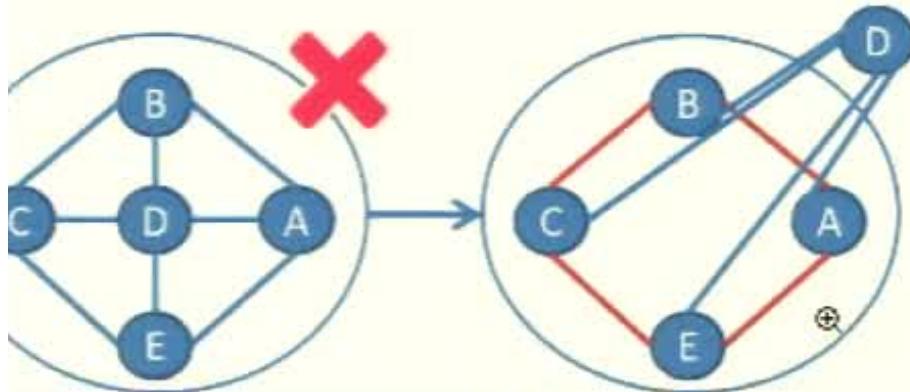
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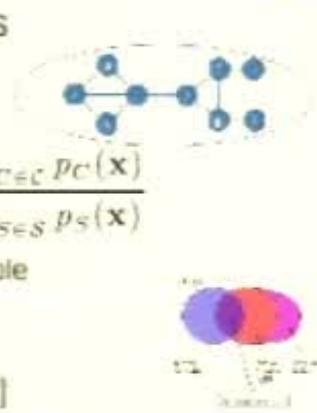
$$p(abcdefghi) = \frac{p(acd) \cdot p(bcd) \cdot p(ce) \cdot p(efg) \cdot p(fh) \cdot p(gi)}{p(cd) \cdot p(c) \cdot p(e) \cdot p(f) \cdot p(g)}$$



Why are decomposable models so interesting?

Properties:

1. Closed form  $\Leftrightarrow p_{\mu}(\mathbf{x}) = \frac{\prod_{C \in \mathcal{C}} p_C(\mathbf{x})}{\prod_{S \in \mathcal{S}} p_S(\mathbf{x})}$
2. Useful:
  - There is always a decomposable model that subsumes a non-decomposable model
3. MLE always exist [Agresti, 2002]
4. Intersection between BN and MRF [Koller, 2009]



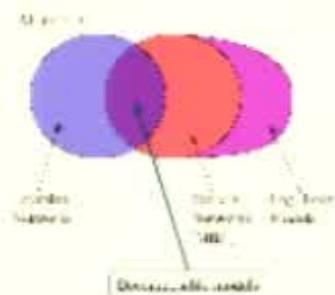
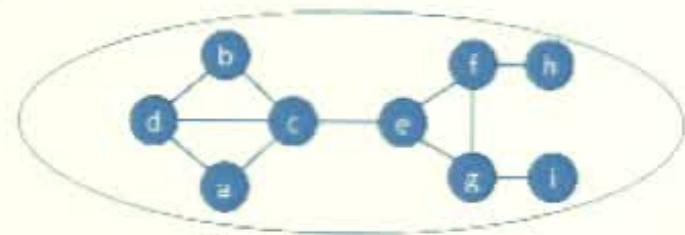
# Why are decomposable models so *interesting*?

## Properties:

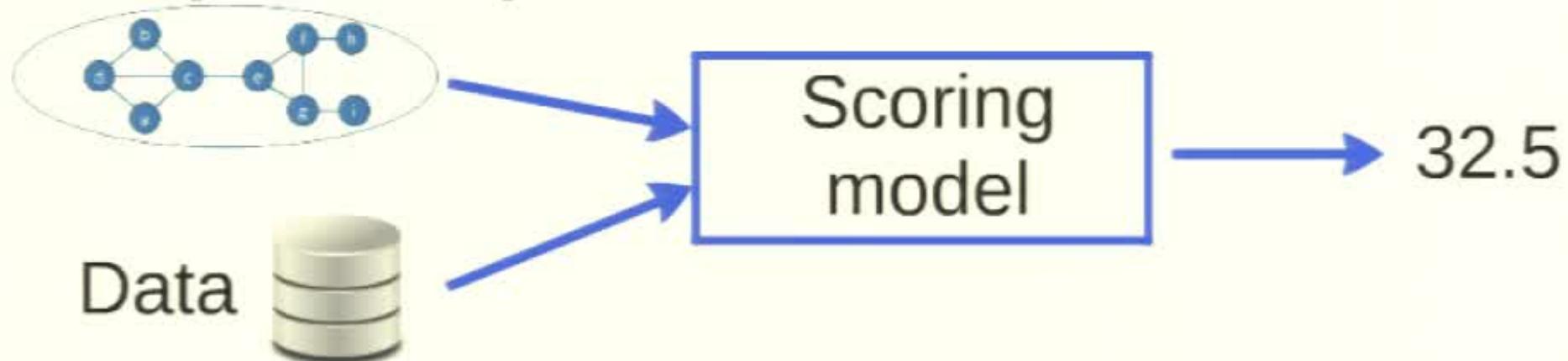
1. Closed form  $\longleftrightarrow p_{\mu}(\mathbf{x}) = \frac{\prod_{C \in C} p_C(\mathbf{x})}{\prod_{S \in S} p_S(\mathbf{x})}$
2. Useful:

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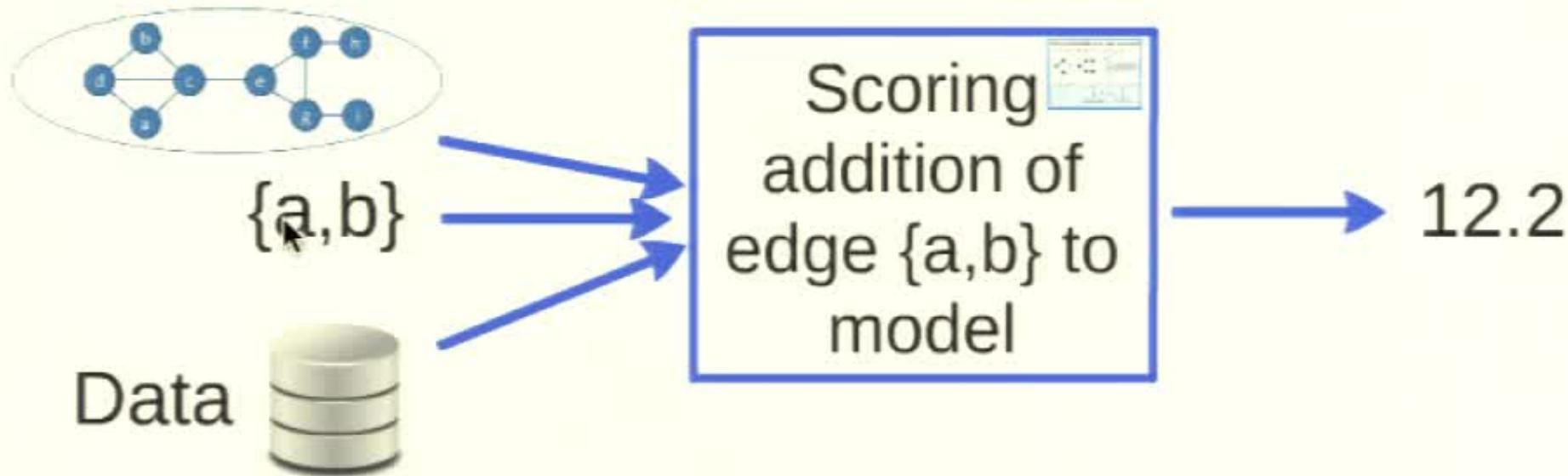
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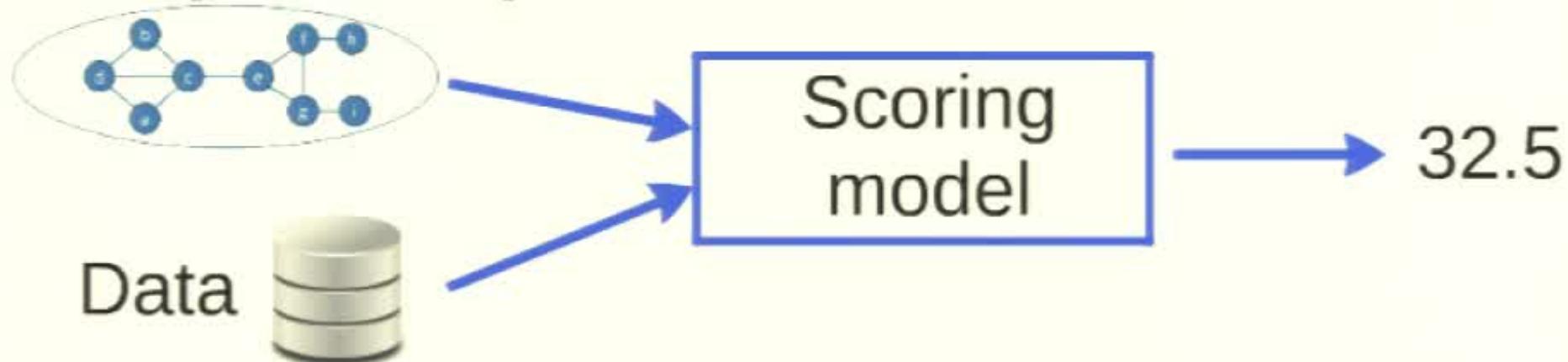
## Scoring decomposable models... ... only relies upon local structures



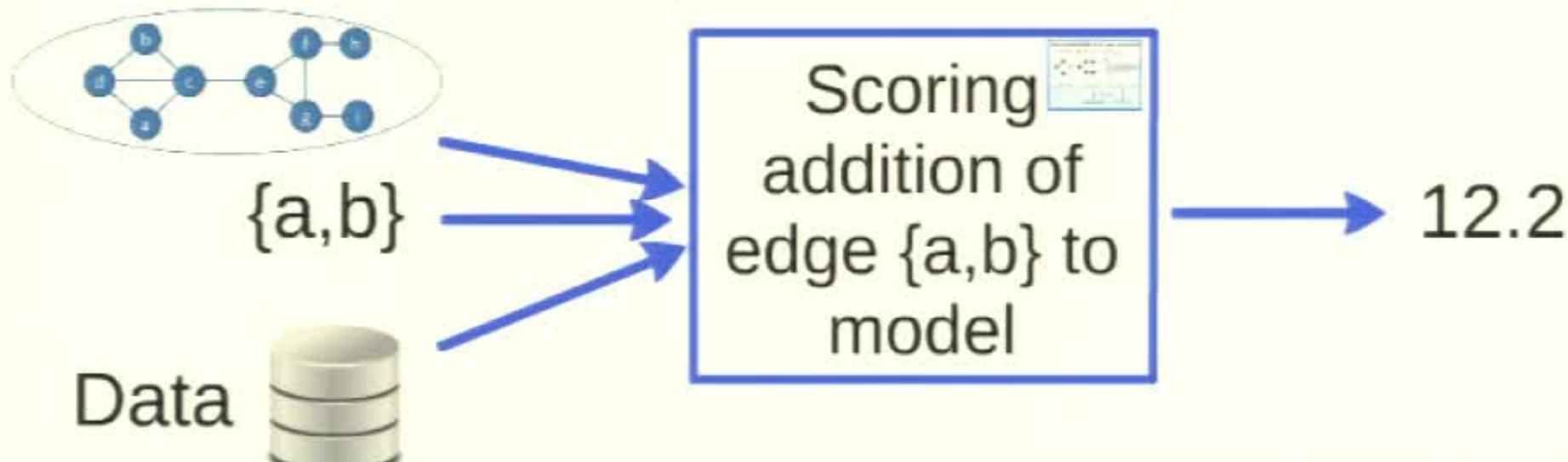
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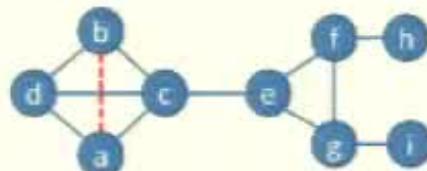
In our case, we only need...



g

### Scoring the addition of an edge to a model

$$\text{score}(\mathcal{M}, (a, b), \mathcal{D}) = \text{score}'(a, b, S_{ab}, \mathcal{D})$$



$S_{ab}$ : minimal separator of (a,b)  
= minimal set of vertices that would disconnect a from b if removed from the graph  
= {c,d}

$$\text{score}(\mathcal{M}, \{a, b\}) = \text{score}'(\{a, b, c, d\}, \{a, c, d\}, \{b, c, d\}, \{c, d\})$$

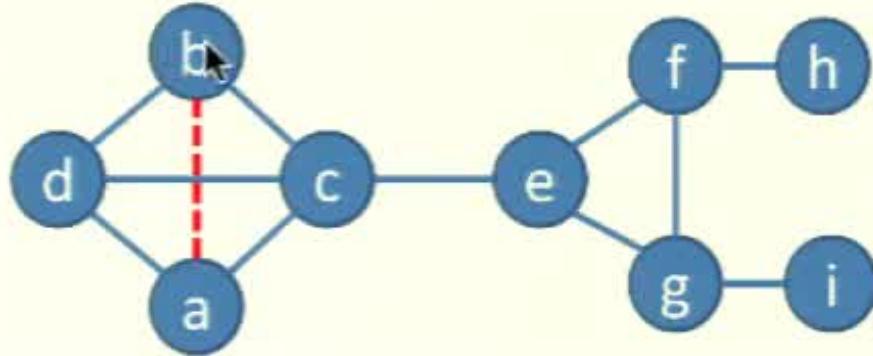


$$S_{ab} \cup a \quad S_{ab} \cup b$$



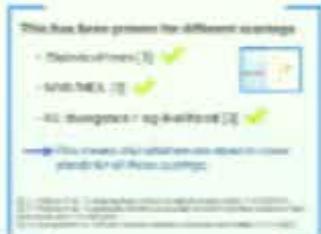
# Scoring the addition of an edge to a model

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= **minimal set of vertices** that would **disconnect** a from b if removed from the graph  
= {c,d}

$$\text{score}(\mathcal{M}, \{a, b\}) = \text{score}'(\{a, b, c, d\}, \{a, c, d\}, \{b, c, d\}, \{c, d\})$$



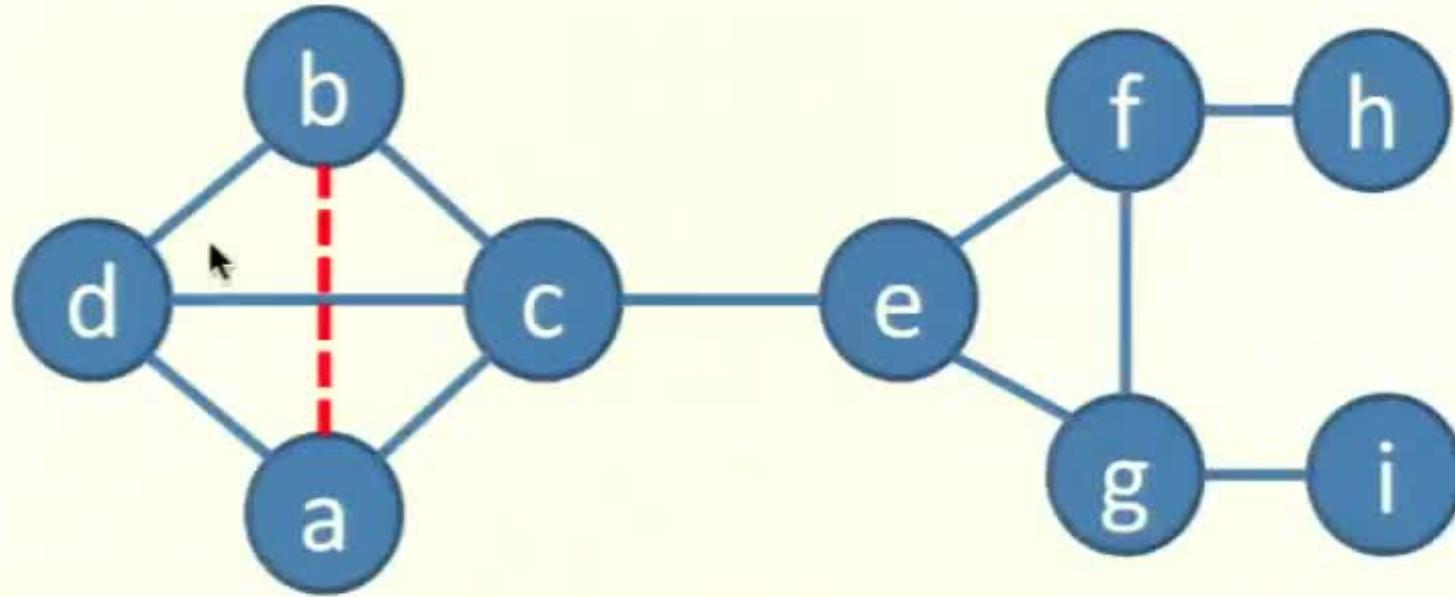
$$S_{ab} \cup a$$

$$S_{ab} \cup a$$

$$S_{ab} \cup b$$

$$S_{ab}$$

$$score(\mathcal{M}, (a, b), \mathcal{D}) = score'(a,$$



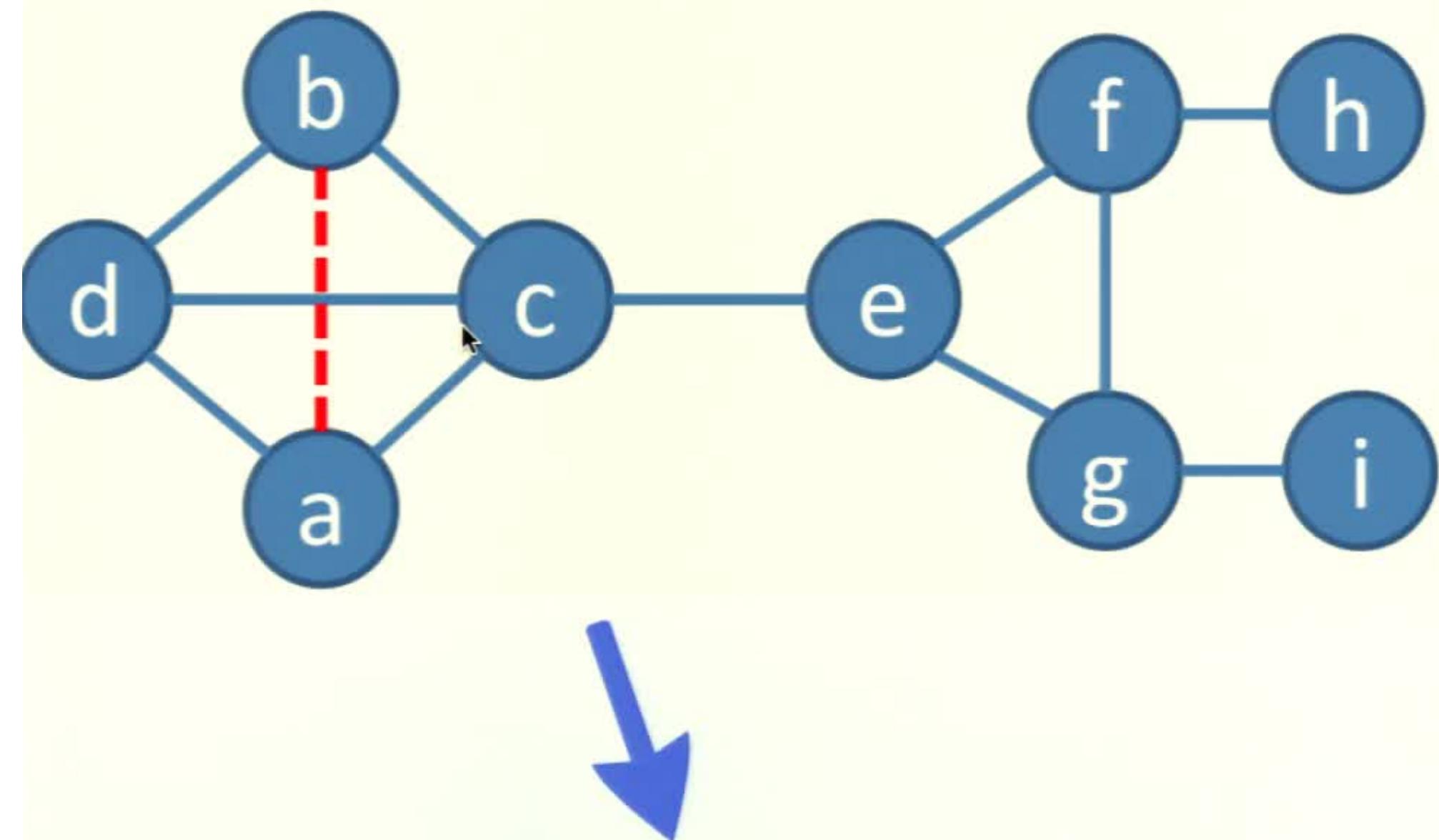
$S_{ab} : n$   
= 1  
word  
= 1

$$score(\mathcal{M}, \{a, b\}) = score'(\{a, b, c, d\}, \{a, b\})$$

This has been proven for different scorings

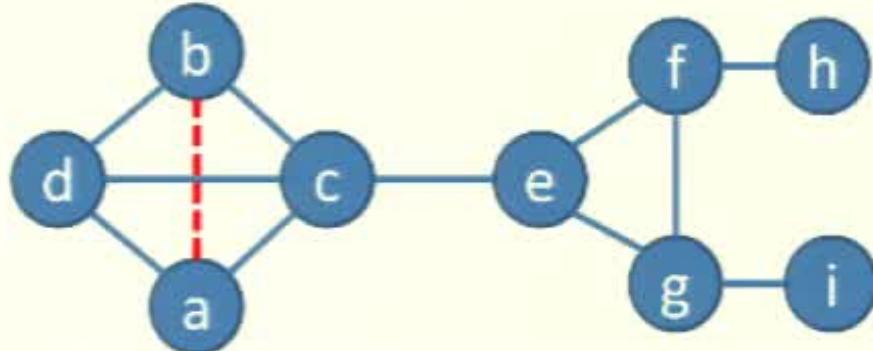
- Statistical tests [1] ✓



$score(\mathcal{M}, (u, v), \mathcal{D}) = sc$ 

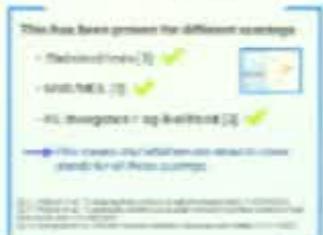
# Scoring the addition of an edge to a model

$$\text{score}(\mathcal{M}, (a, b), \mathcal{D}) = \text{score}'(a, b, S_{ab}, \mathcal{D})$$

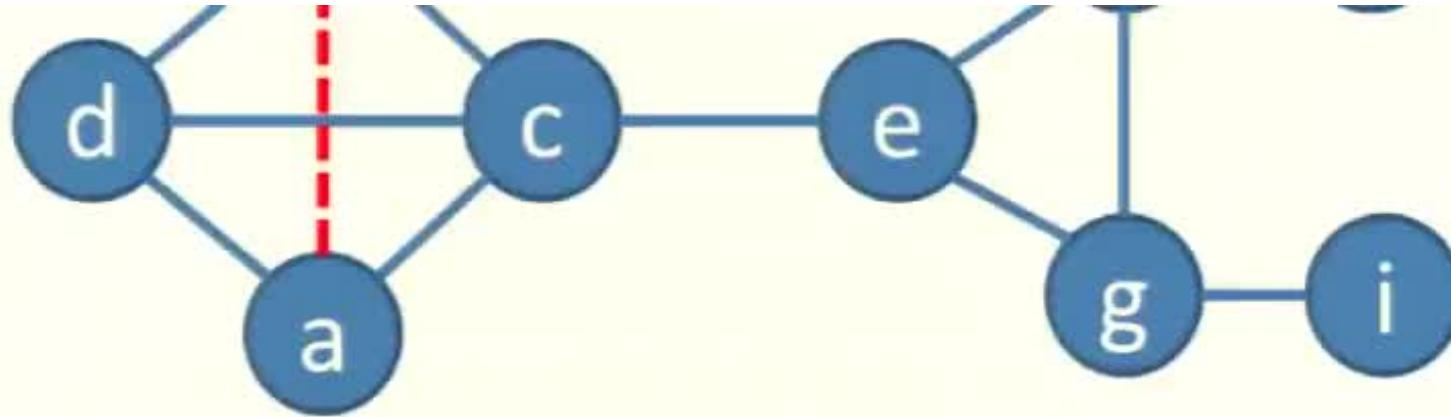


$S_{ab}$ : minimal separator of (a,b)  
= **minimal set of vertices** that would **disconnect** a from b if removed from the graph  
= {c,d}

$$\text{score}(\mathcal{M}, \{a, b\}) = \text{score}'(\{a, b, c, d\}, \{a, c, d\}, \{b, c, d\}, \{c, d\})$$



$$\begin{array}{ccc} S_{ab} \cup a & & S_{ab} \\ \downarrow & & \downarrow \\ S_{ab} \cup a \cup b & & S_{ab} \cup b \end{array}$$



$$score(\mathcal{M}, \{a, b\}) = score'(\{a, b, c, d\} , \{c, e, g, i\})$$

This has been proven for different scorings

- Statistical tests [1] ✓
- MML/MDL [2] ✓
- KL divergence / log-likelihood [3] ✓

→ This means that what we are about to show

is true for all these scorings.

[1] S. Agapiou et al., "Working with sparse high-dimensional data: an ADMM approach".  
[2] P. Fletcher et al., "A statistical efficient and scalable method for surface analysis of high-dimensional data".  
[3] M. Bronstein et al., "Multi-scale convolutional neural networks for point clouds". In CVPR 2017.

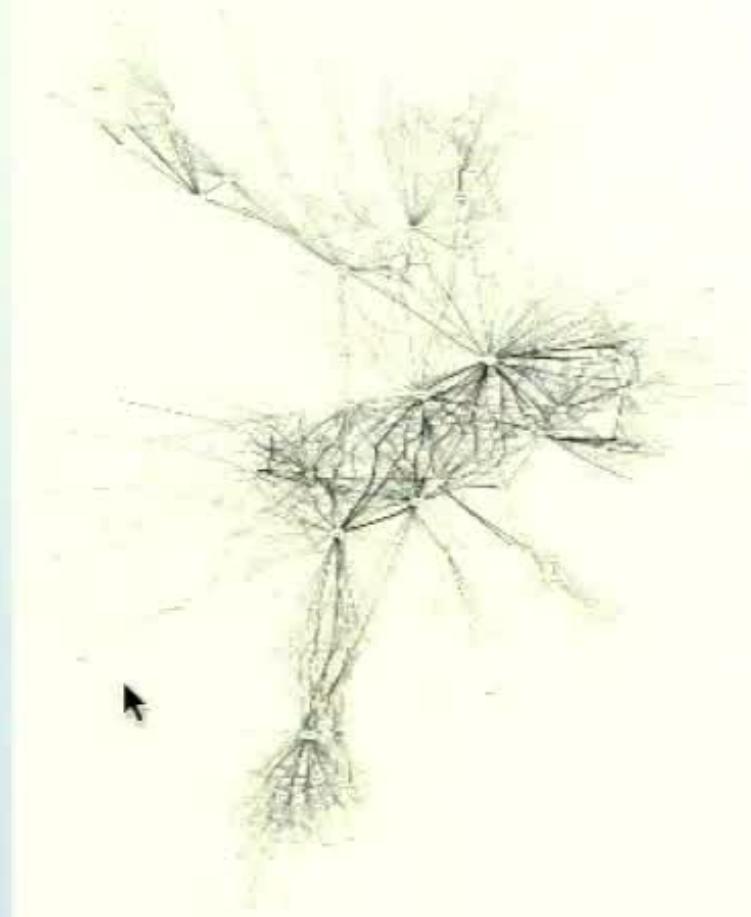


$$S_{ab} \cup a \cup b$$

Assessing the addition of one edge to this model?



We only need to consider **4 cliques**



# This has been proven for different scorings

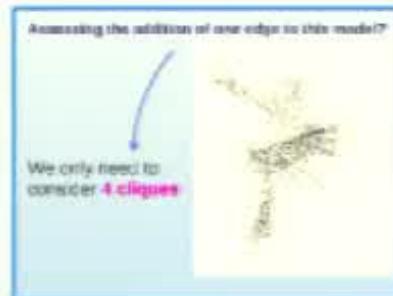
- Statistical tests [1]



- MML/MDL [2]



- KL divergence / log-likelihood [3]



→ *This means that what we are about to show stands for all these scorings.*

[1]: F. Petitjean et al., "Scaling log-linear analysis to high-dimensional data," in *ICDM 2013*.

[2]: F. Petitjean et al., "A statistically efficient and scalable method for log-linear analysis of high-dimensional data," in *ICDM 2014*.

[3]: A. Deshpande et al., "Efficient stepwise selection in decomposable models," in *UAI 2001*.



$score(\mathcal{M}, \{a, b\}) = scc$

This has been proven for different scorings

- Statistical tests [1] ✓



- MML/MDL [2] ✓

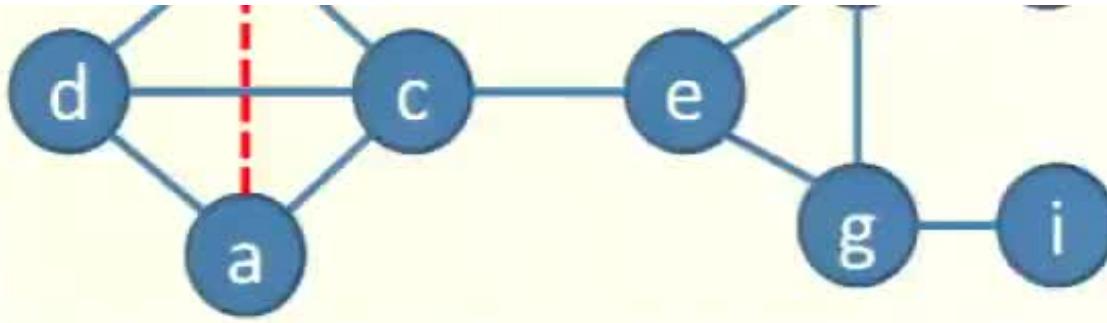
- KL divergence / log-likelihood [3] ✓

→ This means that what we are about to show stands for all these scorings.

[1] F. Petrone et al., "Scaling log-linear analysis to high-dimensional data," in ICDM 2013.

[2] F. Petrone et al., "A statistically efficient and scalable method for log-linear analysis of high-dimensional data," in ICDM 2014.

[3] A. Deshpande et al., "Efficient stepwise selection in decomposable models," in UAI 2001.



$$score(\mathcal{M}, \{a, b\}) = score'(\{a, b, c, d\} ,$$

This has been proven for different scorings

- statistical tests [1] ✓
- ARI, AICL [2] ✓
- ✓ MI, divergence / log-likelihood [3] ✓
- This means that references are about to prove consistency of those scorings.

$$S_{ab} \cup a \cup b$$

# String

models...  
structures  
string  
del

→ 32.5

...  
string  
portion of  
 $\{a,b\}$  to  
odel

→ 12.2

## Intuition



What we have seen :

- Evaluating the addition upon 4 cliques of the

Our intuition

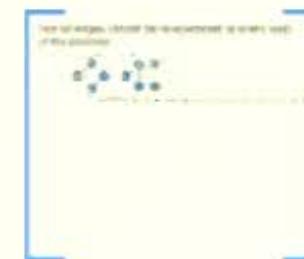
How often does that

How can we use this

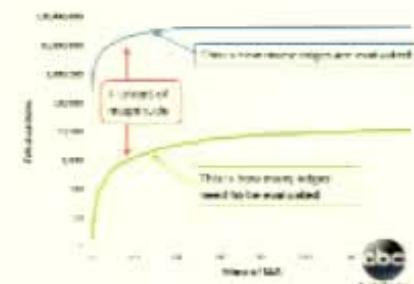
# What we have seen so far:

- Evaluating the addition of an edge only depends upon 4 cliques of the graph

Our intuition



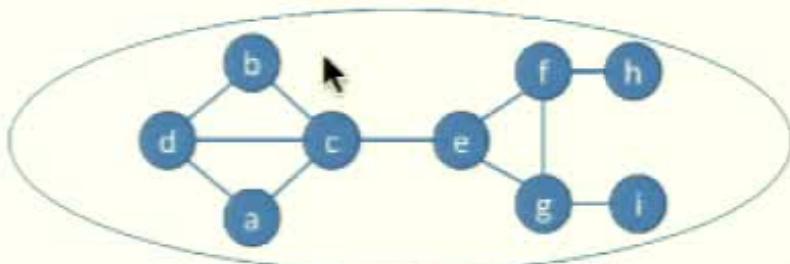
How often does that happen?



How can we use this information?

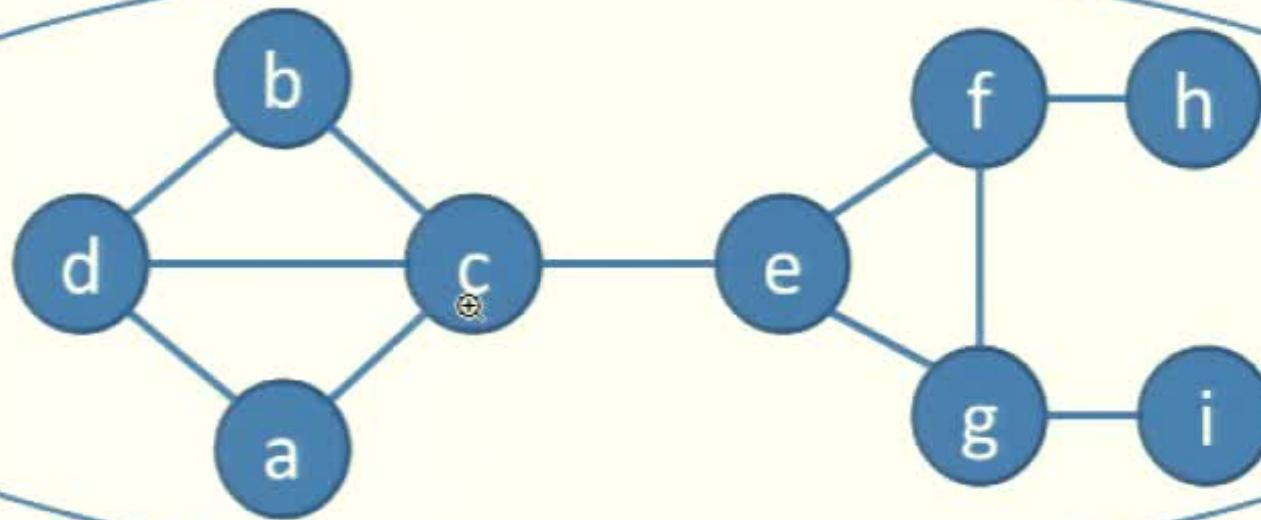


Not all edges should be re-examined at every step of the process



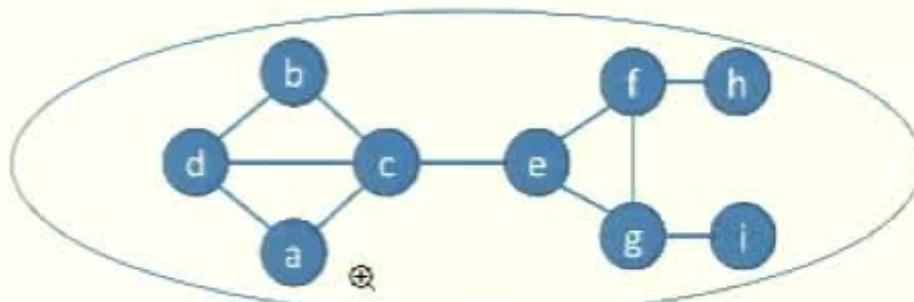
$$score(\mathcal{M}, \{a, b\}) = score'(\{a, b, c, d\}, \{a, c, d\}, \{b, c, d\}, \{c, d\})$$

# e process



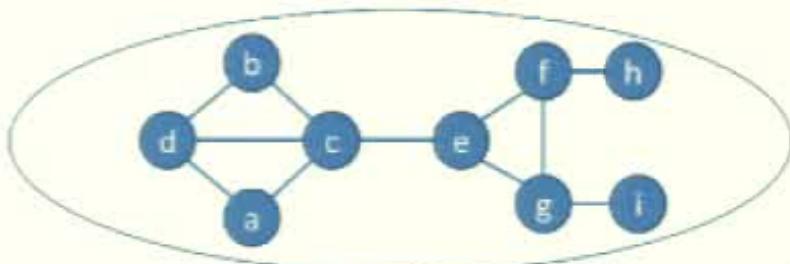
$score(\mathcal{M}, \{a, b\}) = score'(\{a, b, c, d, e, f, g, h\})$

Not all edges should be re-examined at every step of the process



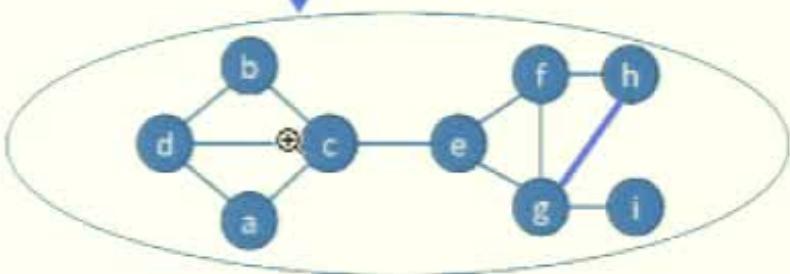
$$score(\mathcal{M}, \{a, b\}) = score'(\{a, b, c, d\}, \{a, c, d\}, \{b, c, d\}, \{c, d\})$$

Not all edges should be re-examined at every step of the process



$$\text{score}(\mathcal{M}, \{a, b\}) = \text{score}'(\{a, b, c, d\}, \{a, c, d\}, \{b, c, d\}, \{c, d\})$$

Select edge {g,h}

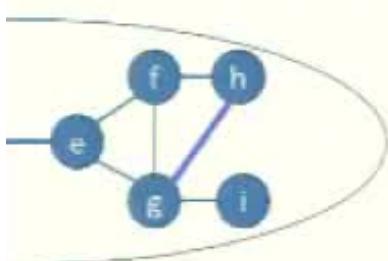


Score({a,b})  
did **not** change

$$\text{score}(\mathcal{M}, \{a, b\}) = \text{score}'(\{a, b, c, d\}, \{a, c, d\}, \{b, c, d\}, \{c, d\})$$

→ *The addition of edge {a,b} need **not** be re-examined in the new model*

}



Score({a,b})  
did **not** change

$$\therefore (\mathcal{M}, \{a, b\}) = score'(\{a, b, c, d\} , \{a, c, d\} , \{b, c, d\} , \{c, d\})$$

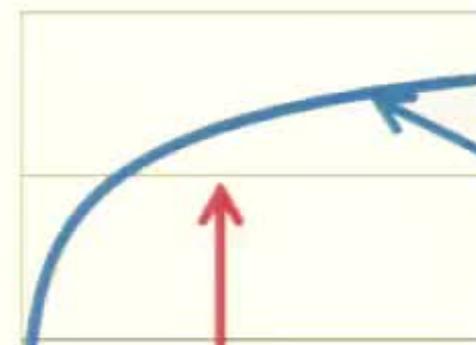
Addition of edge {a,b} need **not** be re-  
fined in the new model

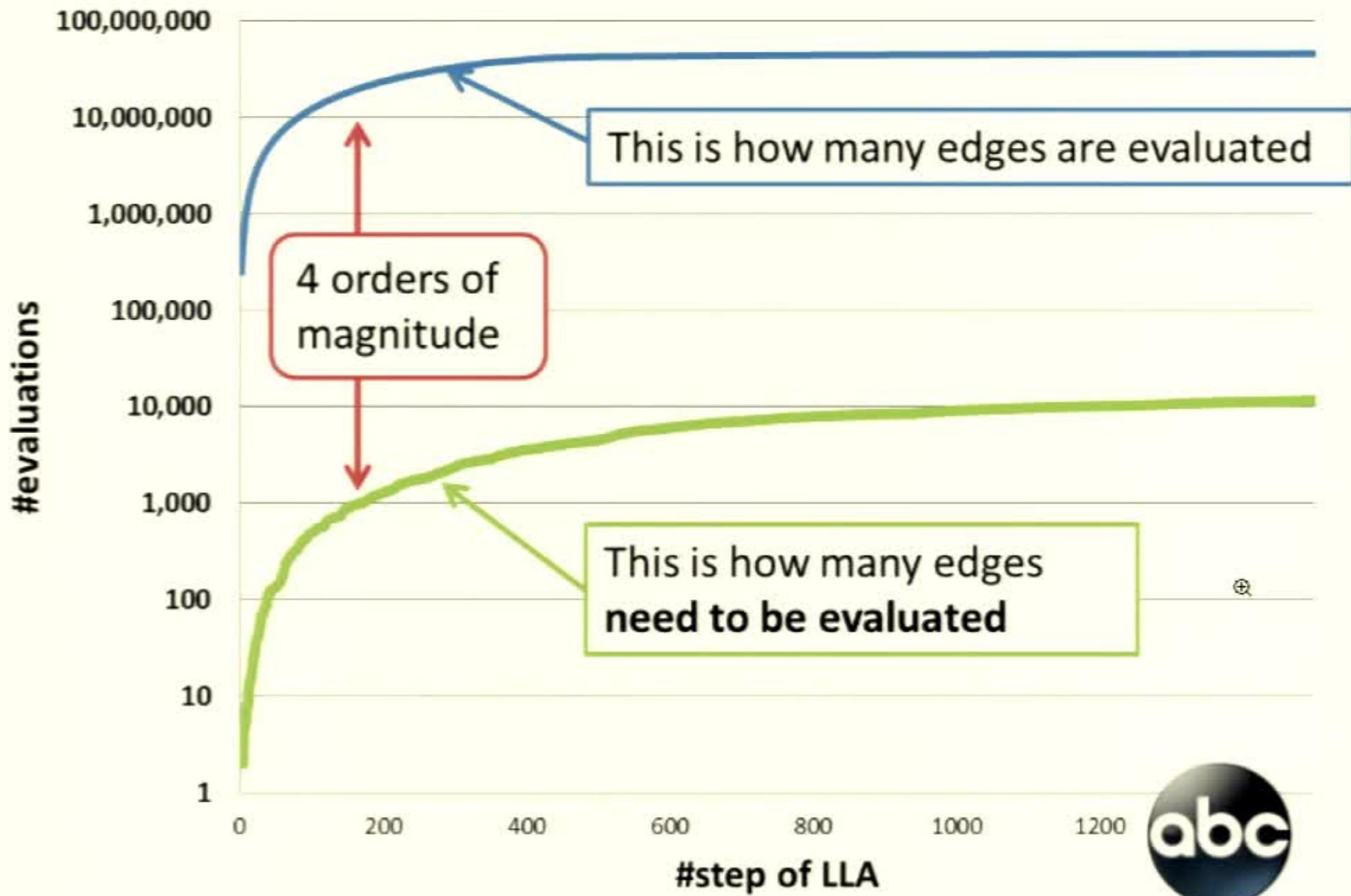


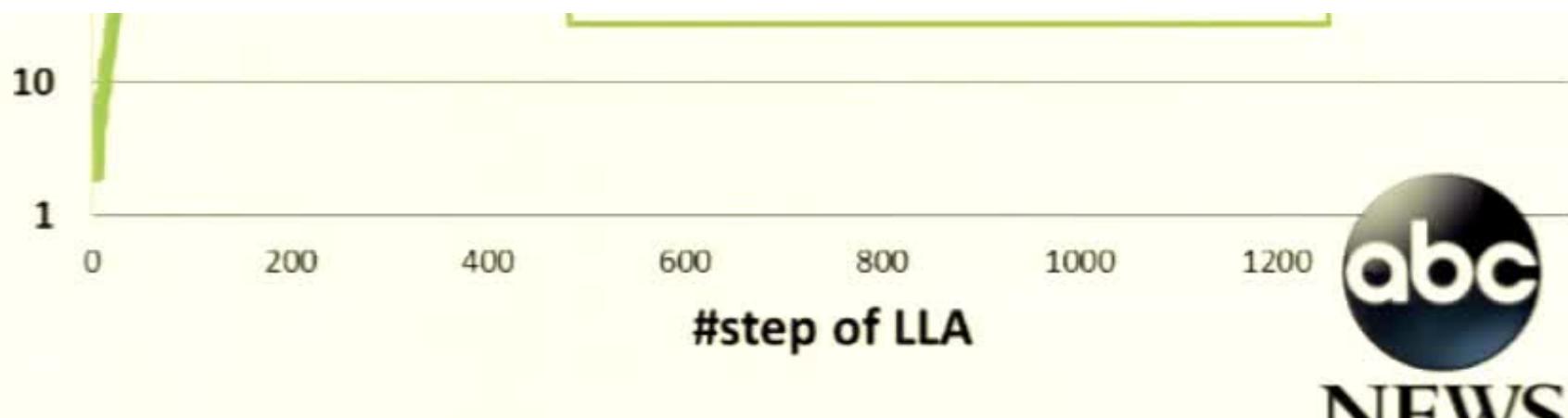
100,000,000

10,000,000

1,000,000





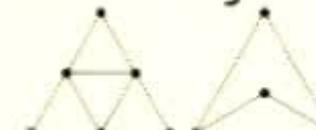


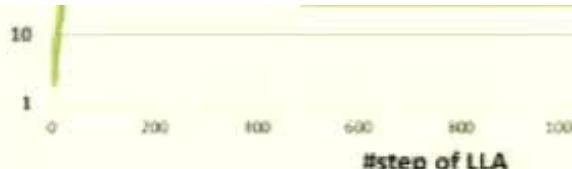
**We know:** if  $S_{ab}$  does not change between different modifications of the graph, then the addition of  $\{a,b\}$  need not be re-examined

1. Use a data structure that gives direct

**We know:** if  $S_{ab}$  does not change between different modifications of the graph, then the addition of {a,b} need not be re-examined



1. Use a data structure that gives direct access to minimal separators for every potential edge A diagram of a complete graph K4 with four vertices and six edges.
2. Keep track of the minimal separators for every potential edge A small electronic device, possibly a smartphone or a specialized mini-computer.
3. Maintain an ordered list of all the potential edges (priority queue) A red stamp with the word "PRIORITY" written diagonally across it.

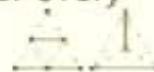


# ion?

We know: if  $S_{ab}$  does not change between different modifications of the graph, then the addition of {a,b} need not be re-examined



1. Use a data structure that gives direct access to minimal separators for every potential edge



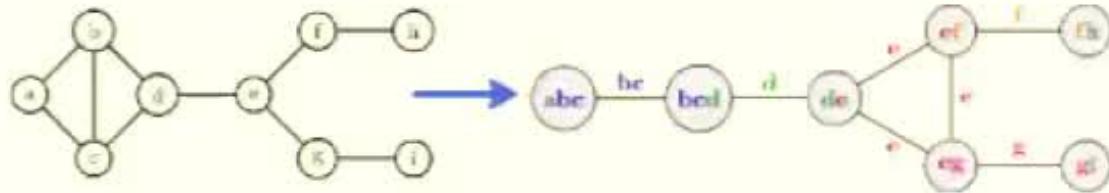
2. Keep track of the minimal separators for every potential edge



3. Maintain an ordered list of all the potential edges (priority queue)

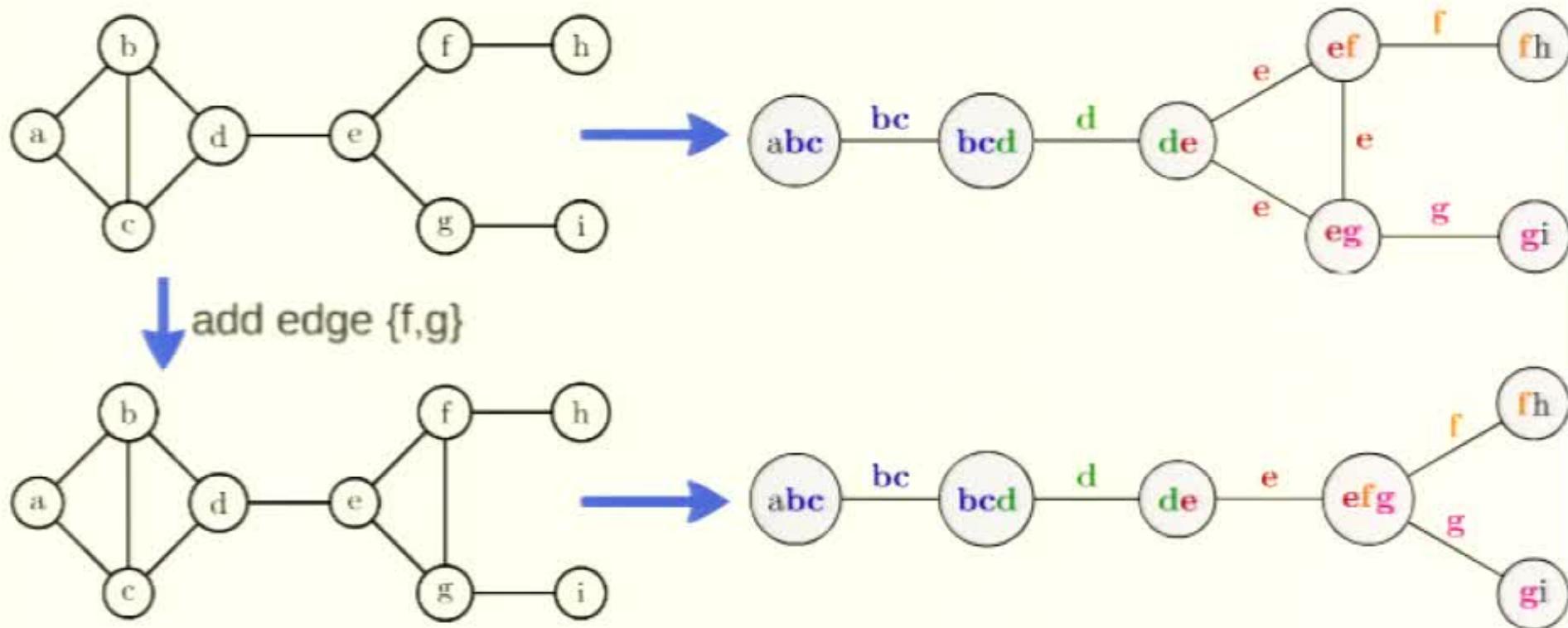


## Clique graph



[1]: A. Deshpande et al., "Efficient stepwise selection in decomposable models," in UAI 2001.

# Clique graph



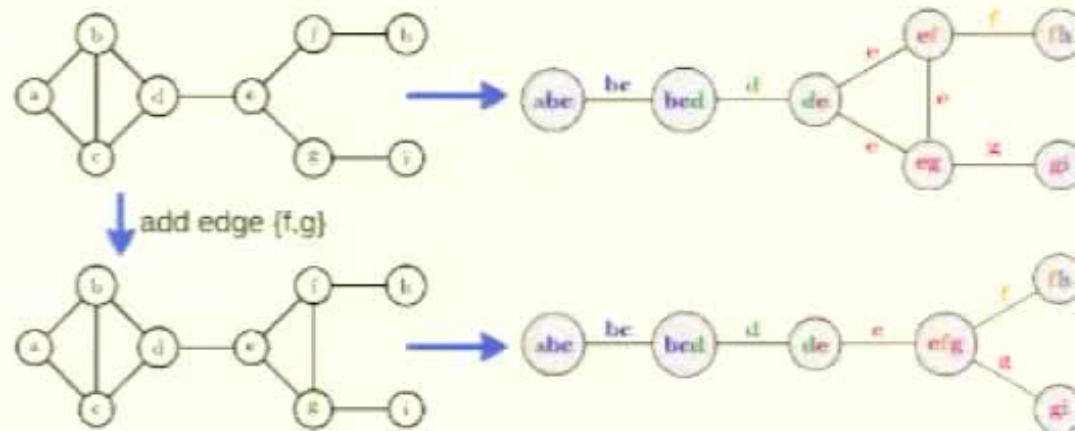
There are algorithms that can directly update the clique-graph [1]

**BUT** those algorithms cannot track the minimal separators  $S_{ab}$   
→ We make this possible - *details in the paper*



[1]: A. Deshpande et al., "Efficient stepwise selection in decomposable models," in UAI 2001.

## Clique graph



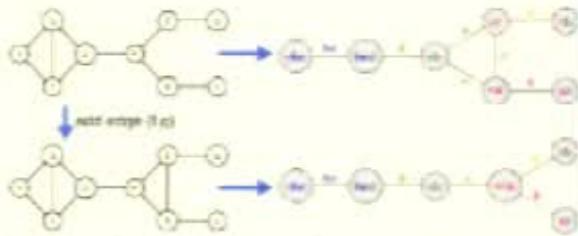
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[1]: A. Deshpande et al., "Efficient stepwise selection in decomposable models," in UAI 2001.

$\gamma, \gamma^2$

## Clique graph

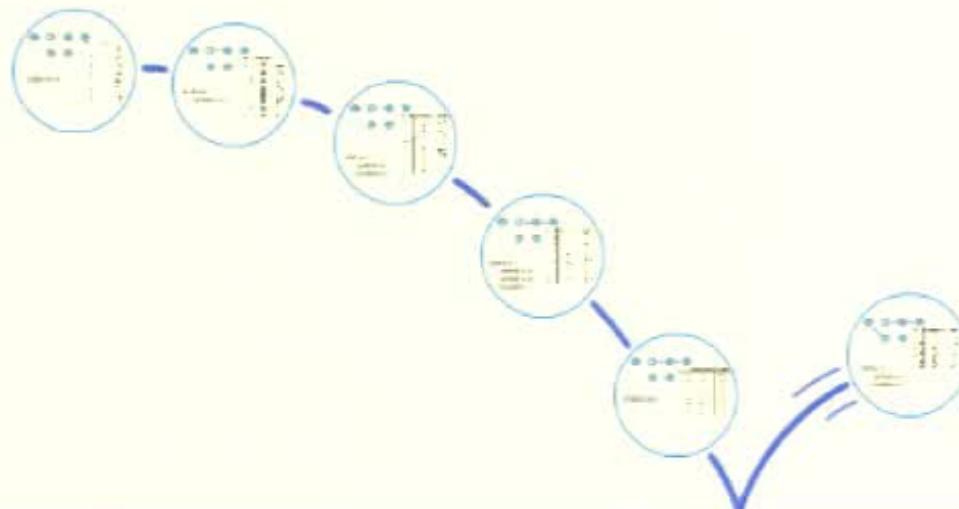


There are algorithms that can directly use the clique graph [1]

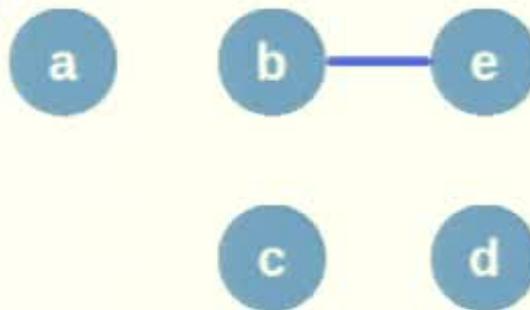
BUT these algorithms cannot track the minimal separators  $S_{ab}$   
→ We make this possible - details in the paper

[1] A. Dobra et al. "Efficient ancestral likelihood in decomposable models." in UAI 2002.

$$v^2 \rightarrow v \cdot \log(v)$$



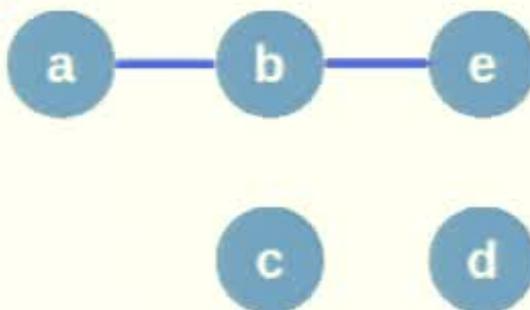
## Priority queue



Add b-e

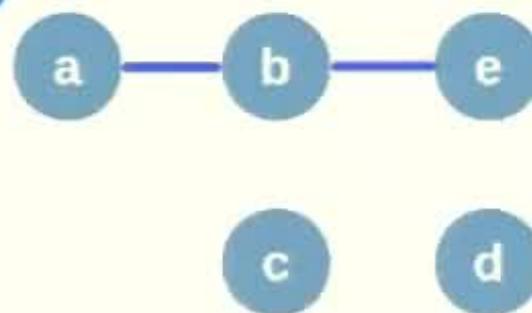
v1	v2	separator	score
b	c	{}	56.2
a	b	{}	72.8
e	f	{}	60.9
a	e	{}	49.5
a	f	{}	42.8
b	c	{}	31.4
c	e	{}	31.0
a	c	{}	28.8
b	f	{}	17.1
c	d	{}	16.9
b	c	{}	12.7
c	f	{}	8.1
d	e	{}	7.3
d	f	{}	4.8
e	f	{}	4.6

Add a-b  
• update a-e



b

v1	v2	separator	score
a	b	{}	72.8
e	f	{}	60.9
a	e	{}	49.5
a	f	{}	42.8
b	c	{}	31.4
c	e	{}	31.0
a	c	{}	28.8
b	f	{}	17.1
c	d	{}	16.9
b	c	{}	12.7
c	f	{}	8.1
d	e	{}	7.3
d	f	{}	4.8
e	f	{}	4.6

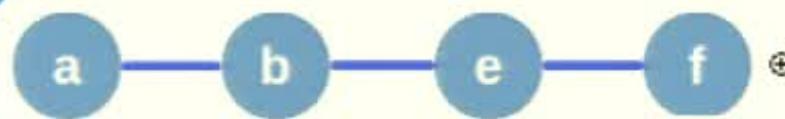


Add e-f

- update b-f
- disable a-f

f

v1	v2	separator	score
e	f	{}{}	60.9
a	f	{}{}	42.8
b	c	{}{}	31.4
c	e	{}{}	31.0
a	c	{}{}	28.8
b	f	{}{}	17.1
c	d	{}{}	16.9
b	c	{}{}	12.7
a	e	{b}{}	12.4
c	f	{}{}	8.1
d	e	{}{}	7.3
d	f	{}{}	4.8
e	f	{}{}	4.6

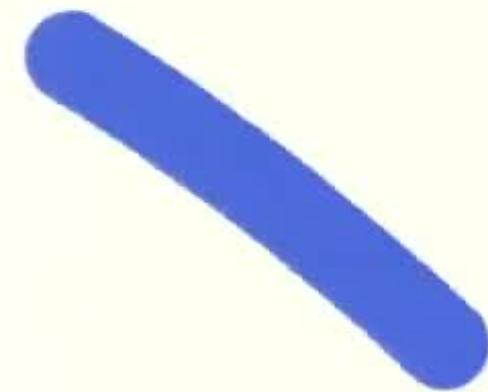


Add e-f

- update b-f
- disable a-f

v1	v2	separator	score
e	f	{}	60.9
a	f	{}	42.8
b	c	{}	31.4
c	e	{}	31.0
a	c	{}	28.8
b	f	{}	17.1
c	d	{}	16.9
b	c	{}	12.7
a	e	{b}	12.4
c	f	{}	8.1
d	e	{}	7.3
d	f	{}	4.8
e	f	{}	4.6

e	{b}	12.4
f	{}	8.1
e	{}	7.3
f	{}	4.8
f	{}	4.6



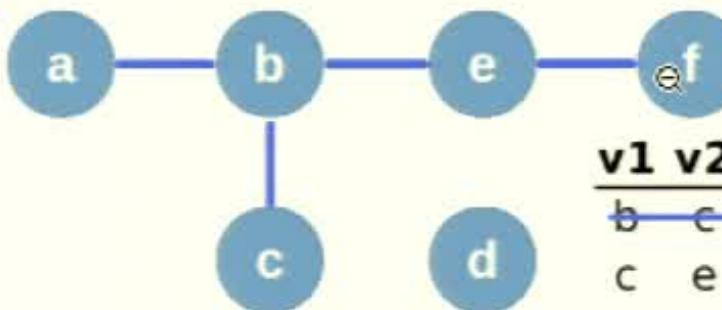
v1	v2	separato
b	c	{}
c	e	{}
a	c	{}
c	d	{}
b	f	{e}
b	c	{}
a	e	{b}
c	f	{}
d	e	{}
d	f	{}

Add b-c

- update a-c
- update c-e

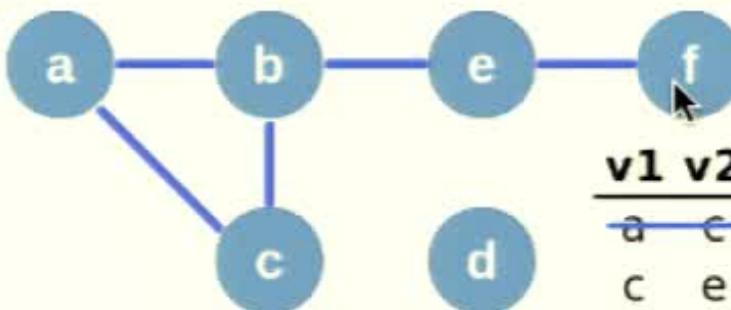
Add b-c

- update a-c
- update c-e
- disable c-f

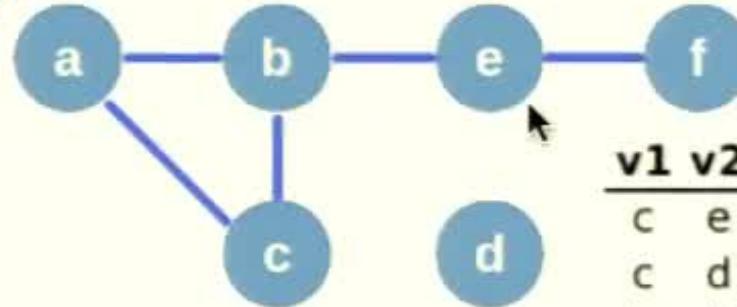


v1	v2	separator	score
b	c	[]	31.4
c	e	{}	31.0
a	c	{}	28.8
c	d	{}	16.9
b	f	{e}	14.0
b	c	{}	12.7
a	e	{b}	12.4
c	f	[]	8.1
d	e	{}	7.3
d	f	{}	4.8
e	f	{}	4.6

Add a-c



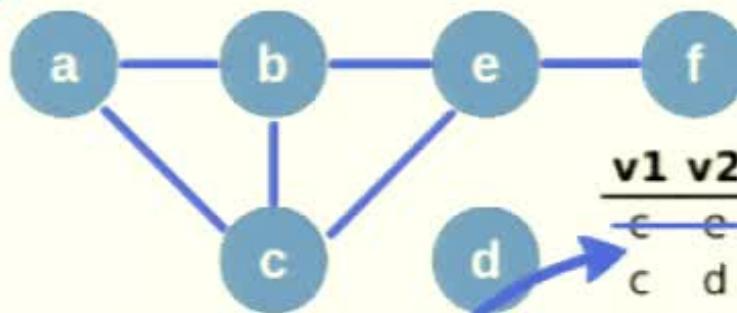
v1	v2	separator	score
a	c	{b}	84.5
c	e	{b}	24.2
c	d	{}	16.9
b	f	{e}	14.0
b	c	{}	12.7
a	e	{b}	12.4
d	e	{}	7.3
d	f	{}	4.8
e	f	{}	4.6



v1	v2	separator	score
c	e	{b}	24.2
c	d	{}	16.9
b	f	{e}	14.0
b	c	{}	12.7
a	e	{b}	12.4
d	e	{}	7.3
d	f	{}	4.8
e	f	{}	4.6

Add c-e

- update a-e
- enable c-f



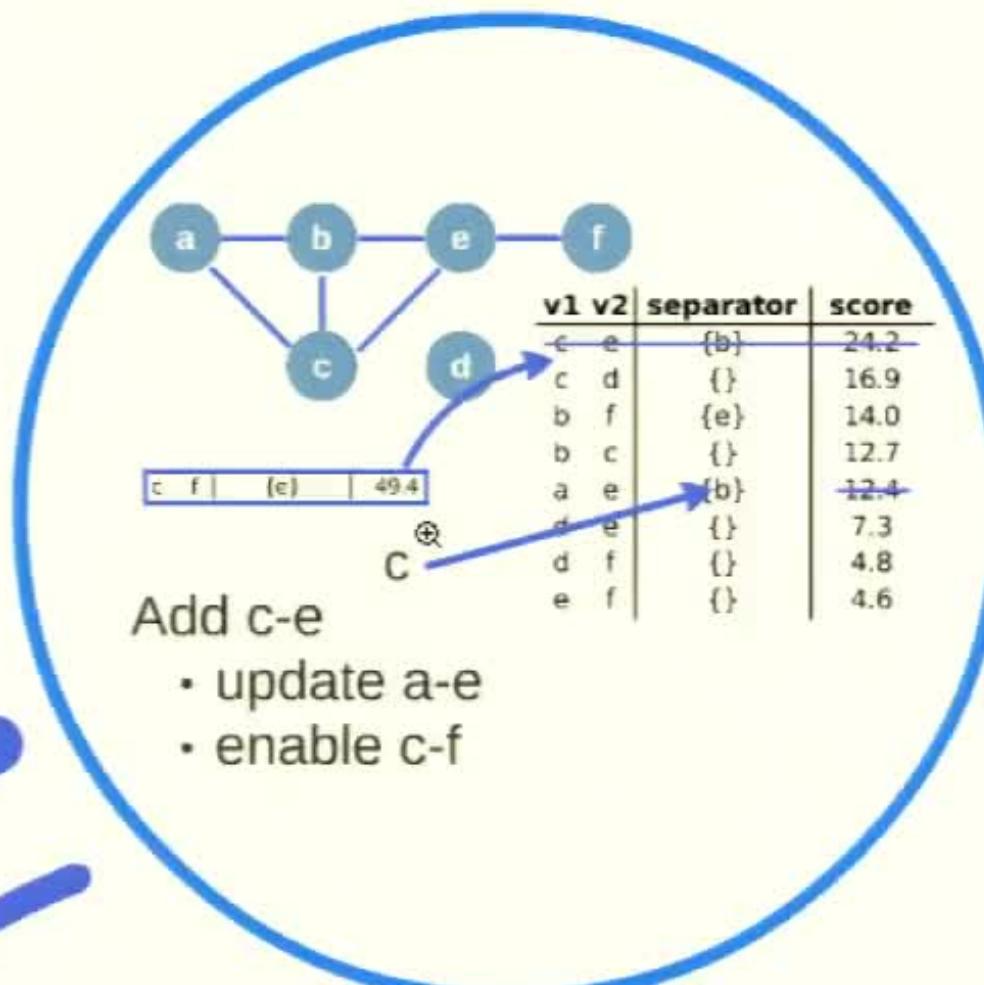
c f | {e} | 49.4

Add c-e

- update a-e
- enable c-f

v1	v2	separator	score
c	c	{b}	24.2
c	d	{}	16.9
b	f	{e}	14.0
b	c	{}	12.7
a	e	{b}	12.4
d	e	{}	7.3
d	f	{}	4.8
e	f	{}	4.6

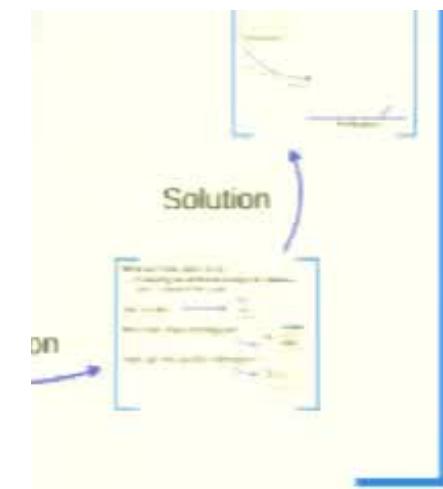
C<sup>⊕</sup>



v1	v2	separator	score
a	b	{}	24.2
a	c	{c}	16.0
a	e	{e}	14.0
a	f	{f}	12.7
b	d	{d}	12.4
b	e	{e}	7.3
b	f	{f}	6.8
c	d	{d}	4.6

Add c-e  
• update a-e  
• enable c-f

# Identified Chordalysis



It works!



# Running times



10 days

1 day

1 hour

1 min

1s

Version 1 - ICDM 2013

Version 2 - V1 + clique graph

Version 3 - V2 + keep track separators

Version 4 - Prioritized Chordalysis

Mushroom

EPESE

Internet

CoIL 2000

MIT Face

ABC news

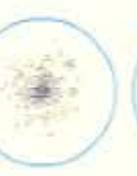
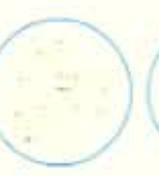
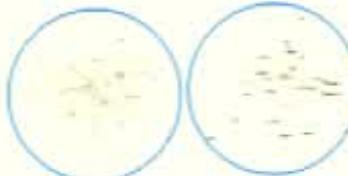
Finance

Protein

Orphamine

NYT

#Vars 20 25 70 85 300 500 500 700 1,200 2,000



# Take-home message



## Prioritized Chordalysis:

1. can analyse data with 1,000+ variables
2. does not sacrifice the soundness
3. is released on **GitHub**



<https://github.com/fpetitjean/>

# Scaling log-linear analysis to datasets with 1,000+ variables

François Petitjean and Geoff Webb



## Thanks for your attention!



<http://www.francois-petitjean.com>



francois.petitjean@monash.edu



@LeDataMiner



# Scaling log-linear analysis to datasets with 1,000+ variables

François Petitjean and Geoff Webb



2015 SIAM International  
Conference on **DATA MINING**

